THE ELEMENTS OF PHYSICS

VOL. II

ELECTRICITY AND MAGNETISM

THE

ELEMENTS OF PHYSICS

A COLLEGE TEXT-BOOK

BY

EDWARD L. NICHOLS

AND

WILLIAM S. FRANKLIN

IN THREE VOLUMES.

VOL. II

ELECTRICITY AND MAGNETISM

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PREFACE.

This edition of the second volume of Nichols' and Franklin's *Elements of Physics* has been entirely rewritten. The traditional treatment of electrostatics, beginning with electrostatic attraction and the definition of the electrostatic unit of charge, has been discarded. It seems better to approach this subject from the standpoint of the ballistic galvanometer inasmuch as, when so developed, the theory of electrostatics is a logical continuation of the foregoing theory of the electric current. Most students begin electrical theory at both ends and never reach the middle.

The student who studies faithfully the first fourteen chapters of this text will profit greatly by a careful study of Chapters XV., XVI. and XVII.

A list of problems covering, as much as possible, the entire subject matter of the text is given at the end of the book.

The thanks of the authors are due to Professor Ernest Merritt, to Professor P. A. Lambert and to Dr. J. S. Shearer for many valuable suggestions, and to Mr. C. M. Crawford for assistance in the preparation of manuscript and in the reading of proof.

South Bethlehem, Pa.,
July 11, 1901.

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THE ELEMENTS OF PHYSICS

VOLUME II.

CHAPTER I.

MAGNETS AND MAGNETIC FIELDS.

1. The magnet.—The name magnet was originally applied to the lodestone, a mineral composed of iron oxide which, in its native state, possesses the power of attracting iron. The lodestone imparts its magnetic property to pieces of iron or steel which are brought near to it. Such pieces of iron or steel are said to be *magnetized* and they are also called magnets. The methods at present employed for magnetizing iron or steel are described in articles 18 and 19.

Poles of a magnet.—Certain parts only of a magnet possess the power of attracting iron. These parts of a magnet are called its *poles*. The poles of a bar magnet are usually at its ends.

Compass. Naming of poles.—A horizontal magnet free to turn about a vertical axis places itself, at most places on the earth, approximately north and south. This fact is made use of in the compass, which consists of a pivoted magnet playing over a divided circle. The terms *magnetic north, magnetic east, etc.*, are occasionally used in referring to the cardinal points of the compass as indicated by the compass needle.

The north-pointing pole of a magnet is called its *north pole*, and the south-pointing pole of a magnet is called its *south pole*.

Mutual action of two magnets.—The north pole of one magnet attracts the south pole of another magnet, and the north poles of two magnets or the south poles of two magnets repel each other; that is, unlike magnetic poles attract each other and like magnetic poles repel each other.

2. Distributed and concentrated poles.—The poles of a magnet, that is, the seats of the attracting or repelling forces, are distributed over considerable portions of the bar, generally the end portions. This is particularly the case with short, thick bars. In the case of long, thin magnets the poles are ordinarily more nearly concentrated at the ends of the bar. In the former case the poles are said to be *distributed*, in the latter case the poles are said to be (approximately) *concentrated*.

The laws of attraction and repulsion of magnets are especially simple for long, thin magnets, that is, for magnets with concentrated poles, and the following discussion applies primarily to such magnets; the various definitions and statements apply, however, to magnets with distributed poles although rigorous statements are in this case very complex.

3. Strength of pole.—The poles of a magnet may attract iron with greater or less force according to the size of the magnet, and according to the thoroughness with which the magnet has been magnetized. The poles of a magnet are said to be *strong* when they attract iron or steel with relatively great force.

Unit pole. Measure of pole strength.—A magnet pole is said to have unit strength, or to be a unit pole, when it exerts an attraction or a repulsion of one dyne upon an equal * pole at a distance of one centimeter, and the force in dynes with which a unit pole acts upon a given pole at a distance of one centimeter, is adopted as the numerical measure of the strength of the given pole.

Remark.—A magnet of m' units strength at a distance of one centimeter from a pole of unit strength is attracted or repelled with a force of m' dynes, according to the above statement. If

* For the purpose of this definition the poles of two entirely similar magnets which have been similarly magnetized, may be considered to be equal.

the pole of unit strength is replaced by a pole of m'' units strength the force action becomes m'' times as great, or $m'm''$ dynes. That is a pole of m' units strength at a distance of one centimeter from another pole of m'' units strength is attracted or repelled with a force of $m'm''$ dynes.

4. Coulomb's law.—*The force of attraction or repulsion of two magnet poles is inversely proportional to the square of the distance between them.* This fact was established experimentally by Coulomb in 1800. This investigator measured the force of attraction of two magnet poles at different distances apart and found the force to vary inversely with the square of the distance. A long, slim magnet was suspended horizontally by a wire, thus forming a torsion pendulum. One of the poles of another slim magnet brought near to one of the poles of the suspended magnet caused a twist of the suspending wire and from the observed value of this twist the force action was calculated as explained in the first volume of this text-book.

Complete expression for the force of attraction of two magnet poles.—According to the previous article two poles attract or repel each other with a force of $m'm''$ dynes when they are one centimeter apart, therefore, according to Coulomb's law, the poles attract with a force of $\frac{m'm''}{r^2}$ dynes when they are r centimeters apart; that is:

$$F = \frac{m'm''}{r^2} \qquad (1)$$

in which m' and m'' are the respective strengths of two magnet poles, r is their distance apart in centimeters, and F is the force in dynes with which they attract or repel each other.

Algebraic sign of magnet pole.—The poles m' and m'' are alike in sign when both are north poles or when both are south poles. On the other hand m' and m'' are unlike in sign when one is a north pole and the other is a south pole. It is customary to consider a north pole as *positive* and a south pole as *nega-*

tive. The force F in equation (1) is considered positive when it is a repulsion.

The two poles of a magnet are always equal in strength, though opposite in sign, as explained in Article 13. When a magnet is broken in two, each piece is a complete magnet with a north pole and a south pole. A bar of steel may be irregularly magnetized so as to have one or more north poles and one or more south poles, some of the poles being near the middle portion of the bar. In such a case the sum total of north polarity is equal to the sum total of south polarity. It is often convenient to speak of an *isolated magnet pole,* meaning one pole of a very long magnet, the other pole being so far away as to be negligible in its effects.

5. Magnetic field.—A magnetic field is any region in which a magnet pole if present is acted upon by a force pulling it in one direction or another. For example, the region surrounding a magnet is a magnetic field; the region surrounding a wire carrying an electric current is a magnetic field. The behavior of a compass needle shows that the entire region surrounding the earth is a magnetic field. The cause of this field is not known.

Direction of a magnetic field at a point.—The force with which a field acts upon a north pole placed at a given point in the field is opposite in direction to the force with which the field at the same point acts upon a south pole. The direction of the force with which a field acts upon a north pole is adopted conventionally as the direction of the field at the point. A magnetic field generally varies in direction (as well as in intensity) from point to point.

6. Intensity of magnetic field.—A magnetic field is said to be *intense* when it exerts a relatively great force upon a given magnet pole.

The force, f, in dynes which acts upon a unit magnet pole when it is placed at a given point in a magnetic field is adopted as the numerical measure of the intensity of the field at the point. This force-per-unit-pole, f, is hereafter spoken of simply as the intensity of the field.

Complete expression for the force with which a magnetic field acts upon a magnet pole.—The force with which a magnetic field acts upon a magnet pole of m units strength is m times as great as the force f with which the field acts upon a unit pole placed at the same point, therefore

$$F = mf \tag{2}$$

in which F is the force in dynes which acts upon a magnet pole of strength m when it is placed in a magnetic field of intensity f.

Homogeneous fields.—A magnetic field is said to be *homogeneous* or *uniform* when it has at every point the same direction (and intensity); otherwise it is said to be *nonhomogeneous* or *nonuniform.* The earth's magnetic field is in many places sensibly homogeneous throughout a room. The magnetic field surrounding a wire carrying an electric current, or the magnetic field surrounding a magnet, is nonhomogeneous.

7. Direction and intensity of the magnetic field surrounding an isolated magnet pole.—A magnet pole of strength m' at a distance r from another pole of strength m'' is, according to equation (1), acted upon by the force $F = \frac{m'm''}{r^2}\left(= \frac{m''}{r^2} \times m'\right)$; but the force acting on m' is, according to equation (2), equal to the product of m' into the field intensity at m' due to m''. Therefore $\frac{m''}{r^2}$ is the intensity of the magnetic field at m' due to m'' and, in general,

$$f = \frac{m}{r^2} \tag{3}$$

in which f is the intensity of the magnetic field at distance r from an isolated magnet pole of strength m.

In the neighborhood of a north pole the magnetic field is directed away from the pole; and in the neighborhood of a south pole the magnetic field is directed towards the pole. This is evident when we consider that the direction of a field is indicated by the direction of the force with which the field acts on a north pole.

8. Representation of magnetic field intensity at a point by means of a line.—The magnetic field intensity at a point, like the velocity of a fluid at a point, may be represented by a line drawn in the direction of the field at the point, and of such length as to represent the intensity of the field at the point to any convenient scale.

9. Composition of magnetic fields.—Consider two agents which, acting *singly*, produce magnetic fields whose intensities at a point p are represented by the lines 1 and 2 in Fig. 1, respectively. These two agents acting *together* produce a magnetic field whose intensity at p is represented by the line 3, which is the resultant of 1 and 2.

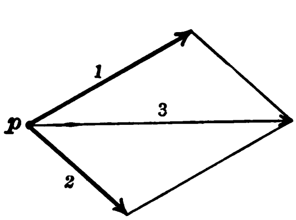


Fig. 1.

A magnetic field due to the combined action of two or more distinct agents is called a *composite field.*

10. Resolution of a magnetic field into components.—Consider a magnetic field whose intensity at a point p, Fig. 2, is represented by the line R. It is often convenient to consider that part of the field which acts in a given direction, a horizontal direction, for example; thus H, Fig. 2, is called the horizontal component of R, and V is called the vertical component of R.

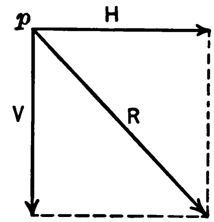


Fig. 2.

11. Lines of force in a magnetic field. Magnetic figures.—A line drawn through a magnetic field so as to be at each point in the direction of the field at that point, is called a *line of force.*

The trend of the lines of force in the neighborhood of a magnet is very strikingly shown by placing a pane of glass over the magnet and dusting iron filings upon it. The filings, becoming magnetized, tend to arrange themselves in filaments along the lines of force. Slight tapping of the glass facilitates the arrangement of the filings. Figs. 3 to 9, and 16, 21, and 24 are photographic reproductions of magnetic figures obtained in this way.

The trend of the lines of force in a magnetic field indicates not

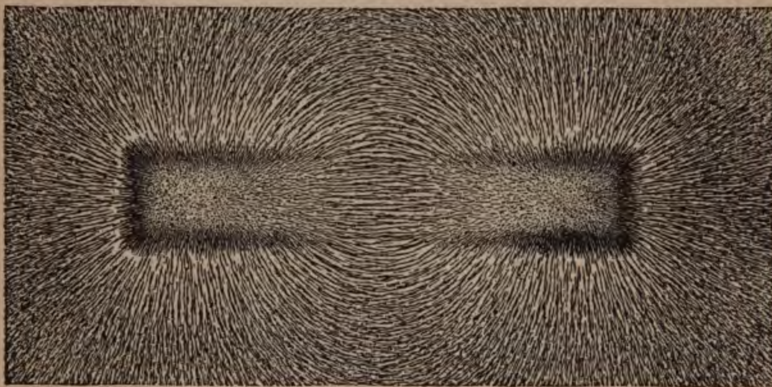


Fig. 3.

only the direction of the field at each point, but also its intensity. In those regions where the lines of force crowd together the field is intense, and in those regions where the lines of force spread

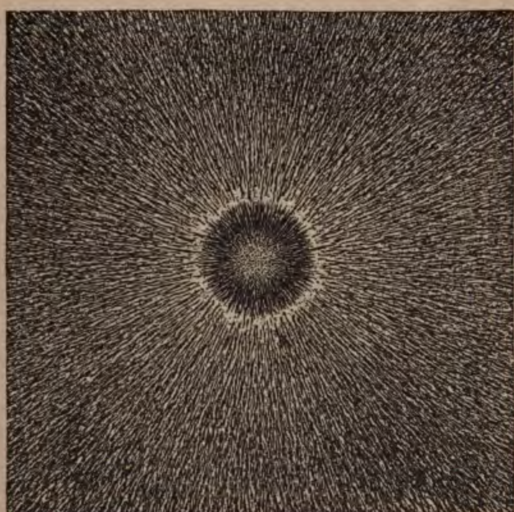


Fig. 4.

apart the field is weak. The lines of force in a uniform field are parallel straight lines.

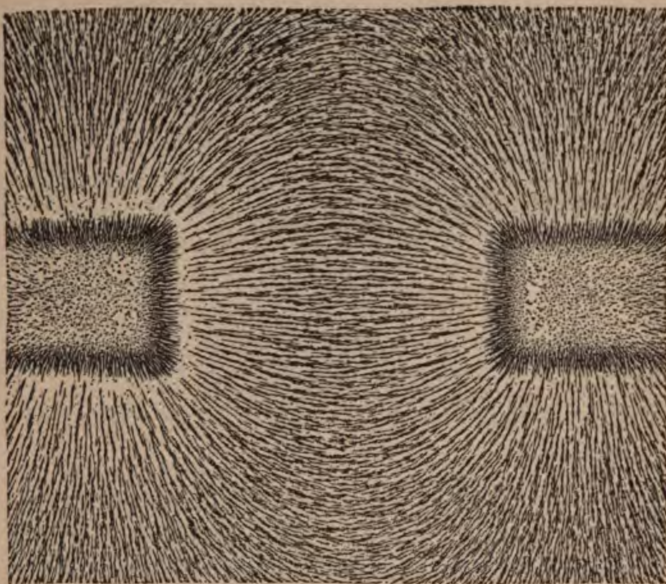


Fig. 5.

Examples of magnetic fields.—(*a*) Fig. 3 shows the trend of the lines of force in the neighborhood of a bar magnet.

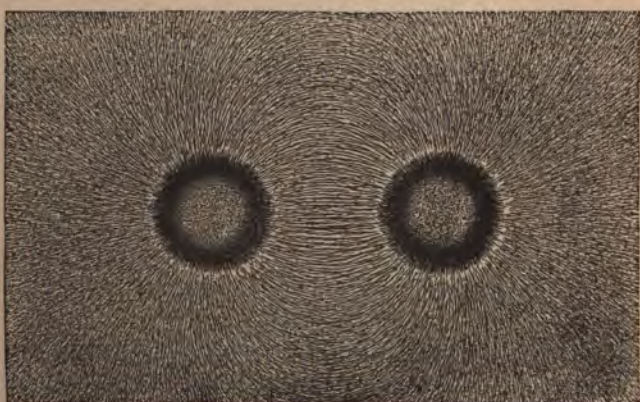


Fig. 6.

(*b*) Fig. 4 shows the trend of the lines of force near the pole of a long slim magnet. The magnet is held in a vertical posi-

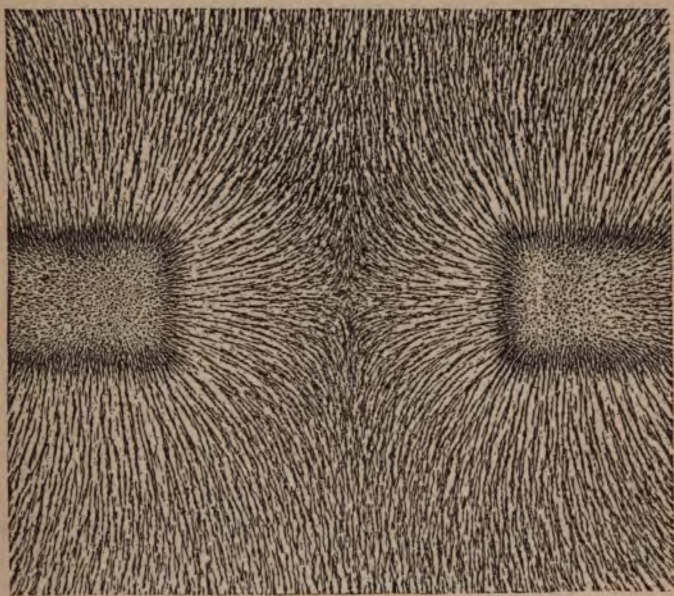


Fig. 7.

tion and the pane of glass, upon which the iron filings are dusted, is laid horizontally upon the end of the magnet.

(*c*) Figs. 5 and 6 show the trend of the lines of force in the neighborhood of *unlike* poles of two magnets.

Fig. 8.

Fig. 9.

(*d*) Fig. 7 shows the trend of the lines of force in the neighborhood of *like* poles of two magnets.

(*e*) Fig. 8 shows the trend of the lines of force near a flat-ended magnet pole.

(*f*) Fig. 9 shows the trend of the lines of force near a pointed magnet pole.

12. Axis of a magnet. Magnetic moment.—The line joining the two poles of a magnet is called the *axis* of the magnet.

The product of the strength of one of the poles of a magnet into the distance between the poles, is called the magnetic moment of the magnet. That is,

$$M = ml \tag{4}$$

in which m is the strength of one of the poles of a magnet, M is the magnetic moment of the magnet, and l is the distance between the poles. The length of a bar magnet is somewhat greater than the distance l of equation (4), inasmuch as the poles are more or less distributed over the ends of the bar.

13. Behavior of a magnet in a uniform magnetic field. Numerical equality of the two poles of a magnet.—The two forces with which a uniform magnetic field acts upon the two poles of a magnet tend in general to turn the magnet but they do not tend to impart to the magnet a motion of translation. These two forces are therefore equal in magnitude and opposite in direction.

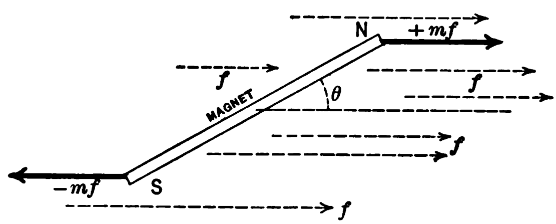


Fig. 10.

The numerical equality of the forces with which a uniform field s upon the two poles of a magnet shows that *the poles of a* *magnet are numerically equal in strength*, but of course opposite in sign.

Discussion.—Consider a magnet of length l, placed in a uniform magnetic field of intensity f; the angle between the axis of the magnet and the direction of the field being θ, as shown in Fig. 10. The poles of the magnet are acted upon by the forces $+mf$ and $-mf$ respectively. The torque action of each of these forces about the center of the magnet is $mf \times \frac{l}{2} \sin \theta$. Therefore the total torque, T, tending to turn the magnet into the direction of the field is

$$T = -mlf \sin \theta \tag{5}$$

The negative sign is chosen for the reason that the torque tends to reduce θ, which is a positive angle. When $\theta = 0$, or when $\theta = 180°$, this torque is zero. A magnet is in stable equilibrium when its north pole points in the direction of a magnetic field (θ equal to zero), and in unstable equilibrium when its south pole points in the direction of a magnetic field (θ equal to $180°$).

If θ is very small, then θ (in radians) may be written for $\sin \theta$ in equation (5), giving

$$T = -mlf \cdot \theta \tag{6}$$

This equation shows (see discussion of harmonic motion in Vol. I., pages 49 and 72) that a suspended magnet, when started, will vibrate about its axis of suspension in such a manner that

$$\frac{4\pi^2 K}{\tau^2} = Mf \tag{7}$$

in which K is the moment of inertia of the magnet about the axis of suspension, τ is the period of one vibration, and $M (= ml)$ is the magnetic moment of the magnet. Equation (7), depending as it does upon equation (6), is not true if θ reaches a large value.

14. Gauss's method for measuring the horizontal component, *H*, of the earth's magnetic field, and for measuring the magnetic moment of a magnet.

First arrangement.—A large magnet, whose magnetic moment is *M*, is suspended horizontally at the place where *H* is to be determined, and set vibrating about the vertical axis of suspension. From equation (7), writing *H* for *f*, we have ·

$$\frac{4\pi^2 K}{\tau^2} = MH \tag{i}$$

Second arrangement.—A small magnet *NS* (Fig. 11) is suspended at the place occupied by the large magnet in the first arrangement. This small magnet, being free to turn, points in the direction of the magnetic field in which it is placed, that is, in the direction of *H*. The large magnet is now placed with its center at a distance *d* due magnetic east or west of *NS*, as shown in Fig. 11. This large magnet then produces at *NS* a magnetic field *h*, at right angles to *H*. The small magnet now points in the direction of the resultant of *h* and *H*, having turned through an angle ϕ. From the diagram (Fig. 11) we have:

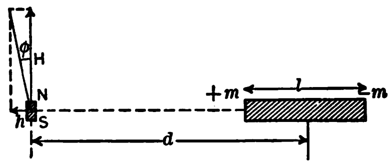


Fig. 11.

$$\tan\phi = \frac{h}{H} \tag{ii}$$

From equation (3) we have $\dfrac{-m}{\left(d - \dfrac{l}{2}\right)^2}$ as the expression for the intensity of the magnetic field at *NS* due to the pole $-m$ of the large magnet; and $\dfrac{+m}{\left(d + \dfrac{l}{2}\right)^2}$ for the field intensity at *NS* due to the pole $+m$; so that:

$$h = \frac{m}{\left(d - \frac{l}{2}\right)^2} - \frac{m}{\left(d + \frac{l}{2}\right)^2} \qquad \text{(iii)}$$

This equation may be simplified as follows: Reduce the fractions $\frac{m}{\left(d - \frac{l}{2}\right)^2}$ and $\frac{m}{\left(d + \frac{l}{2}\right)^2}$ to a common denominator and substitute M for ml in accordance with equation (4). We then have

$$h = 2Md \frac{1}{\left(d^2 - \frac{l^2}{4}\right)^2}$$

Multiply numerator and denominator of the second member of this equation by $\left(d^2 + \frac{l^2}{4}\right)^2$ and we have:

$$h = 2Md \frac{d^4 + \frac{d^2 l^2}{2} + \frac{l^4}{16}}{\left(d^4 - \frac{l^4}{16}\right)^2}$$

In this expression $\frac{l^4}{16}$ may be dropped, since l is small compared to d, and l^4 is very small compared to d^4. Therefore:

$$h = \frac{2M}{d^5}\left(d^2 + \frac{l^2}{2}\right) \qquad \text{(iv)}$$

Substitute this simplified value of h in equation (ii) and we have:

$$\tan \phi = \frac{2M}{Hd^5}\left(d^2 + \frac{l^2}{2}\right) \qquad \text{(v)}$$

The large magnet may now be placed nearer to NS (Fig. 11), say at distance d_1, and the corresponding angle of deflection being ϕ_1, we have

$$\tan \phi_1 = \frac{2M}{Hd_1^5}\left(d_1^2 + \frac{l^2}{2}\right) \qquad \text{(vi)}$$

The uncertain quantity l, which is the distance between the poles of the large magnet, may be eliminated from equation (v) with the help of equation (vi), giving:

$$\frac{M}{H} = \frac{d^5 \tan \phi - d_1^5 \tan \phi_1}{2(d^2 - d_1^2)} \qquad \text{(vii)}$$

Observations and calculations.—The quantity τ, equation (i), is observed and K is calculated from the measured mass and dimensions of the large magnet, leaving only M and H unknown in equation (i). The quantities d, d_1, ϕ, and ϕ_1 in equation (vii) are observed, leaving only M and H unknown in (vii). Equation (i) and (vii) then enable the calculation of both M and H.

If it is desired to determine the strength of the poles of the large magnet, the quantity l may be approximately measured, and m calculated from the equation $M = ml$.

This method * for determining M and H was devised by Gauss.

15. The behavior of a magnet in a nonuniform magnetic field.—The forces which act on the poles of a magnet which is placed in a *nonuniform magnetic field* tend in general to turn the magnet and also *to impart to it a motion of translation.* This latter tendency is due to the fact that the force which acts on the north pole of the magnet is in general not opposite in direction and not equal in value to the force which acts on the south pole of the magnet. That is, the field at the north pole of the magnet is in general different in intensity and in direction from the field at the south pole of the magnet.

The attraction of a particle of iron by a magnet depends in the first place upon the magnetization of the particle of iron (see Art. 18), and in the second place upon the *nonuniformity* of the magnetic field in which the magnetized particle finds itself. The magnetic field near a flat ended magnet pole is approximately uniform as shown in Fig. 8 except near the corners of the pole.

* For fuller discussion of Gauss' method see A. Gray, "Absolute Measurements in Electricity and Magnetism," Vol. II., p. 69.

Therefore only the corners of a flat ended pole attracts small particles of iron. The magnetic field near a pointed magnet pole varies rapidly in intensity from point to point as shown in Fig. 9. Therefore a pointed magnet pole has a strong attraction for small particles of iron.

A pointed magnet pole is an essential feature of the magnetic ore separator, the action of which is shown in Fig. 12. The crushed ore falls in a thin stream before a pointed or wedge-shaped magnet pole. The particles of magnetic material are attracted by the pointed pole and thus deflected while the nonmagnetic material falls straight downwards.

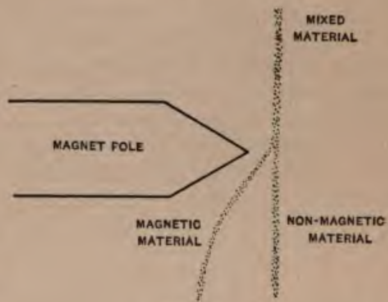


Fig. 12.

A pointed magnet is sometimes used for removing particles of iron or steel from the eye.

16. Magnetic field a state of stress of the ether.—The attraction and repulsion of magnet poles depends upon a stressed condition of the intervening medium, the *ether*.* Thus the ether between two unlike (attracting) poles is in a state of tension. A careful analysis of magnetic attraction and repulsion shows that the ether at a point in a magnetic field is under tension in the direction of the field at the point and under compression in all directions at right angles to this; or, in other words, the lines of force in a magnetic field tend to shorten themselves, and they tend to push each other apart sidewise. Thus, the attraction of unlike poles may be ascribed to the tendency of the lines of force to shorten, see Figs. 5 and 6; the repulsion of like poles may be

* Magnetic fields can exist in a region devoid of ordinary matter; that is, in a vacuum. It is therefore necessary, if one is to reach a mechanical conception of the magnetic field, to assume the existence of a medium, the *ether*, which permeates all space.

ascribed to the tendency of the lines of force to push each other apart sidewise, see Fig. 7.

The mechanical conception of the magnetic field.—The stressed condition of the ether in a magnetic field may be ascribed to a state of motion of the ether; in fact, the ether, at each point in a magnetic field, may be considered to rotate at great angular velocity about an axis which is parallel to the direction of the magnetic field at the point. This type of motion of a fluid (the ether being supposedly a fluid) is called *vortex motion*. Such a motion of the ether would tend to shorten the rotating portions in the direction of the axis and to expand them in all directions at right angles to the axis. Thus the tendency of the lines of force to shorten their length and to push apart sidewise would be explained.

Illustration.—This supposed action of the ether may be roughly illustrated as follows: A short piece of rubber tube is filled with water, its ends are plugged, and it is mounted on an axis as shown in Fig. 13. A rapid rotation of this tube about the axis

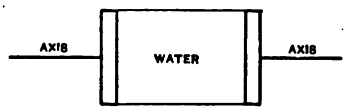


Fig. 13.

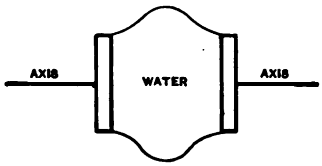


Fig. 14.

causes it to bulge as shown in Fig. 14, and to grow shorter in the direction of the axis of rotation.

17. The magnetic field as a seat of kinetic energy.—To establish a magnetic field work must be done, and work is regained when the field is destroyed. The magnetic field is therefore a seat of energy, and in accordance with the above conception this energy is kinetic. The work done in pulling two unlike magnet poles apart goes to establish the magnetic field which comes into existence between the poles. The work gained when unlike poles are allowed to approach each other comes from the field which is destroyed.

It can be proven * that:

$$W = \frac{1}{8\pi} f^2$$

in which W is the *kinetic energy per unit volume* in a magnetic field, in the neighborhood of a point at which the intensity of field is f.

18. Magnetization.—When a piece of iron, or other magnetic substance, is placed in a magnetic field it becomes a magnet. For example, a neutral or unmagnetized bar of iron or steel when held in the direction of the earth's magnetic field shows north polarity at one end and south polarity at the other. If the

* See J. J. Thomson, "Elementary Lessons in Electricity and Magnetism," pp. 70 and 264.

bar is turned end for end, its magnetism is reversed. A sharp blow with a hammer renders the bar much more susceptible

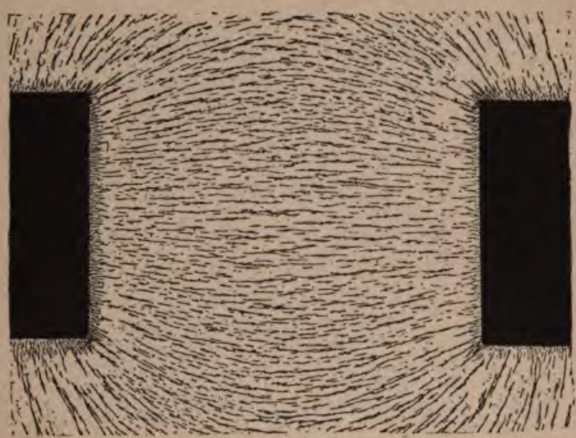


Fig. 15.

to the influence of the weak magnetic field of the earth. The polarity of the bar is easily indicated by a small magnetic needle

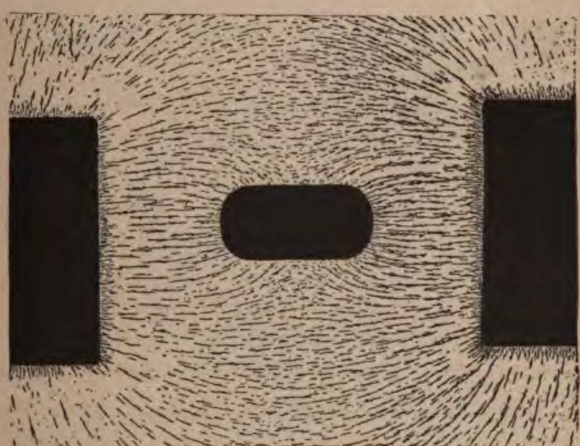


Fig. 16.

hung by a silk fiber or supported by a jewel and point. This action of a magnetic field upon iron is called *magnetization*.

When a piece of iron is placed in a magnetic field the trend of the lines of force in the field is greatly altered; really, the field becomes the resultant of two fields, namely the original field and the field due to the piece of iron which has become a magnet. Thus Fig. 15 shows the effect of a small piece of iron upon the magnetic field between two flat-ended magnet poles. In the absence of the iron the field is as shown in Fig. 16. The effect of a piece of iron in a magnetic field is always such as to suggest that "iron is a better carrier of lines of force than air." The lines of force tend to converge into the iron and pass through it.

Remark.—The region surrounding a magnet is a magnetic field, and magnetizes any piece of iron in the neighborhood. A piece of iron is always magnetized by an adjacent magnet in such a way as to be *attracted* by the magnet.

19. The electromagnet.—The magnetizing action of a magnetic field upon iron is most strikingly shown by the comparatively intense magnetic field inside of a coil of wire carrying an electric current. Thus an iron rod wound with insulated wire becomes a very strong magnet when an electric current is sent through the wire. An iron rod wound with wire in this way is called an *electromagnet.**

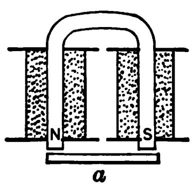


Fig. 17.

A common type of electromagnet is shown in Fig. 17. It consists of a bent iron rod with a winding of wire distributed along it or at its ends. The poles N and S are near together and may both act to attract a strip of iron a, called the *armature*.

20. Residual magnetism. Permanent magnets.—An iron rod retains much of its magnetism when it is removed from a magnetic field in which it has been magnetized; or, in case of an electromagnet, when the magnetizing current is reduced to zero. Long bars retain a greater portion of their magnetism than short

* That end of the rod becomes a north pole, towards which a right-handed screw would move if turned in the direction in which the current circulates around the rod.

bars, because of the fact that in short bars the poles of the magnet are closer together and produce of themselves a strong demagnetizing field along the bar. The magnetism thus left in a bar of iron or in an electromagnet is called *residual magnetism.* Long bars of annealed wrought iron may retain in this way as much as ninety per cent. of their magnetism, but a very weak demagnetizing field or a very slight mechanical shock is sufficient to cause such a bar to lose nearly all of its residual magnetism. Cast iron, hard drawn iron wire, and mild steel retain a smaller portion of their magnetism, but with greater persistence, and hardened steel bars retain a portion of their magnetism very persistently, even when roughly handled. Magnetized bars of hardened steel are called *permanent magnets.* The more persistently a sample of iron retains its magnetism, the greater the intensity of magnetic field needed to magnetize it. Thus hardened steel bars are best magnetized by placing them between the poles of a strong electromagnet, or by placing them inside of a large coil of wire, through which a strong current is sent.

Aging of permanent magnets.—A freshly magnetized bar of hardened steel loses a portion of its residual magnetization readily when subjected to mechanical shocks or to changes of temperature. After the residual magnetization has been reduced in this way, a remainder is left which changes but little with repeated mechanical shocks and changes of temperature, and the magnet is said to be *aged.* Permanent magnets for use in electrical measuring instruments are always subjected to an aging process which usually consists in placing the magnet repeatedly in hot and then in cold water, and in subjecting it to a series of slight mechanical shocks.

Demagnetization.—When iron is heated to bright redness it loses its magnetic properties. Thus red-hot iron is not attracted by a magnet. When a magnetized bar of steel is heated to bright redness its magnetization disappears and upon cooling the bar is found to be completely demagnetized.

A piece of iron or steel may be completely demagnetized by the following operation: The piece is placed in a coil of wire through which a strong electric current is flowing. This current is then repeatedly reversed in direction and at the same time slowly reduced in value until it reaches zero. This operation is called *demagnetization by reversals*; it may also be performed as follows: The piece of steel to be demagnetized is brought near to a strong magnet, set rotating about an axis perpendicular to the field due to the strong magnet, and slowly moved away from the magnet, care being taken to keep the axis of rotation perpendicular to the field. This operation is usually employed in demagnetizing the steel parts of a watch.

21. Intensity of magnetization. Magnetic saturation.—Let m be the strength of the magnetic pole at the end of an iron rod of sectional area q. The ratio $\frac{m}{q}$ is called the *intensity of magnetization*, I, of the rod. That is:

$$I = \frac{m}{q} \tag{8}$$

When an iron rod is subjected to a stronger and stronger magnetizing field its magnetization, I, becomes more and more intense and approaches a definite limiting value beyond which it cannot be magnetized however strong the magnetizing field. The iron rod is said to approach *saturation* as it approaches this limiting intensity of magnetization. For soft wrought iron this limiting value of I is about 1,730 units pole per square centimeter section of rod, for mild steel the limiting value of I is about 1,600, for cobalt it is about 1,310, and for nickel it is about 540.

Permanent magnets of hardened steel have at the utmost an intensity of about 800 units pole per square centimeter section.

22. The compensation of the ship's compass.—In modern iron ships the compass is greatly affected by the magnetism of the ship and the satisfactory use of the compass on such ships depends upon the elimination by compensation of the ship's magnetic action.

The magnetic action of a ship is due in part to *permanent magnetization* (that is a magnetization which does not change as the ship moves about); and in part to a *temporary magnetization* which depends upon the intensity of the earth's magnetic field and its direction with reference to the ship.

The effect of the permanent magnetization of the ship upon the compass may be compensated by placing a permanent steel magnet in such a position near the compass as to produce at the compass a magnetic field equal and opposite to the field produced at the compass by the ship's permanent magnetization.

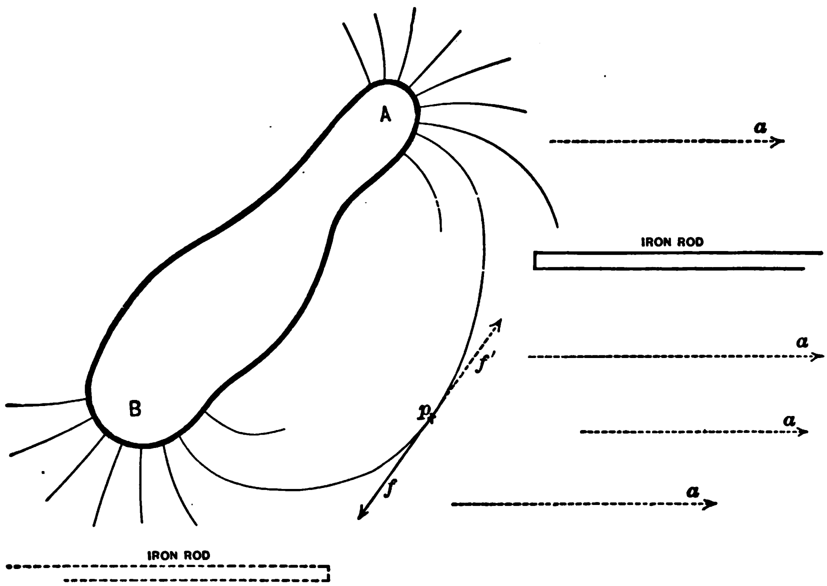


Fig. 18.

In discussing the method for compensating for the effect of the ship's temporary magnetization upon the compass it is convenient to look upon the ship as stationary and to think of the earth's field as changing its direction with reference to the ship.

Let the earth's field be resolved into three rectangular components *a*, *b* and *c*, each component being in a direction fixed with reference to the ship. Consider the effect of one only of these components, say *a*. Let *AB*, Fig. 18, be a mass of iron which represents the ship, and let the point *p* represent the position of the compass. The compass and the iron rod are of course aboard the ship, but they are spread apart in the figure for the sake of clearness. The temporary magnetization of *AB* due to *a* produces in the surrounding region a magnetic field of which the lines of force are represented by the curved lines in Fig. 18, and the intensity of this field at *p* is represented by the arrow *f*. An iron rod parallel to *a* may be so placed that its temporary magnetization due to *a* produces a field of which the intensity at *p* is equal and opposite to *f* as shown by the dotted arrow f'. There are, as shown in the figure, two positions of the rod which satisfy the requirements. If now the component *a* changes in intensity the temporary magnetization of *AB* and of the rod both change in proportion and the field intensities *f* and f' at *p* remain equal and opposite so that the only field at *p* is simply *a*.

Similarly the effect of the temporary magnetization of *AB* due to the component *b* may be compensated at *p* by a properly placed iron rod parallel to *b* and the effect of the temporary magnetization of *AB* due to the component *c* may be compensated at *p* by a properly placed iron rod parallel to *c*. Then the compass placed at *p* will point in the direction of the earth's field; that is, in the direction of the resultant of *a*, *b* and *c*.

In practice the compass of a ship is only partially compensated.

23. The molecular theory of the magnetization of iron.—When a magnet is broken in pieces each piece is found to be a complete magnet having a south pole and a north pole. This fact suggests the possibility that each molecule of iron may be a magnet. Indeed the hypothesis that *each molecule of iron, or of any substance capable of being magnetized, is a permanent magnet* leads to a very useful conception of what takes place in a bar of iron when it is magnetized.

Explanation of magnetization.—In unmagnetized iron or steel the molecular magnets are thought of as pointing at random in all directions, thus neutralizing each other. When the iron or steel is placed in an intense magnetic field the molecular magnets are turned with their axes parallel to the field and the iron or steel is completely magnetized or saturated. If the magnetizing field is weak, the molecular magnets are only partially turned and the iron is only partially magnetized.

Explanation of the retention of magnetization.—When a bar of iron is strongly magnetized it does not return to its initial state when the magnetizing field ceases to act. This is analogous to the production of a *permanent set* when an imperfectly elastic substance is greatly distorted. This persistence of a portion of the magnetization in a strongly magnetized bar may be ascribed to a friction-like opposition to the turning of the molecular magnets when they have been considerably turned. In annealed iron this friction is small, in hard drawn iron wire, cast iron and mild steel it is greater, and in hardened steel it is very great. Mechanical vibration and rise of temperature both act as if to decrease this frictional resistance, enabling a given magnetizing field to produce more intense magnetization and causing residual magnetization to disappear.

The behavior of iron and steel when subjected to weak magnetic fields.—When a bar of iron is placed in a weak magnetic field, or when the magnetic field in which a bar is placed is changed slightly in intensity, then, in general, the magnetization returns to its initial value when the magnetizing field is brought back to its initial value. That is, a bar of iron exhibits a kind of magnetic elasticity, which manifests itself in an opposition to the turning of the molecular magnets such that, if they are not turned too far, they tend to return to their initial configuration when the disturbing cause is removed. This action is especially prominent in hardened steel. Thus a small magnet may be held so as to be repelled by a strong magnet. When brought near to the strong magnet the magnetization of the small magnet is reversed by the intense magnetic field and it is attracted, when removed again to a distance the small magnet may regain its original magnetization and be repelled as at first.

Ewing's theory.—Ewing has shown * that the apparent frictional and elastic opposition to the turning of the molecular magnets may both be ascribed to the *mutual action of these molecules as magnets.* This physicist constructed a model consisting of a number of small magnets supported upon jewels and pivots and arranged on a board. When this system of magnets is subjected to the action of a weak magnetic field each magnet is slightly turned and every magnet returns to its initial position when the weak field ceases to act. If the field is slowly increased in intensity the magnets are turned more and more until the configuration of the system becomes unstable; when the magnets suddenly fall as it were into a new configuration.† If now the field is slowly reduced in intensity the magnets persist in their new configuration until the field intensity reaches a much smaller value than that for which the above mentioned instability occurred.

24. Paramagnetic substances. Diamagnetic substances.—Cobalt and nickel are similar to iron in their magnetic properties except that the limiting or saturation value of their intensity of magnetization is not so great. Many other substances, such as manganese, chromium, platinum, oxygen, many iron compounds, etc., show similar properties but to a less degree. Such substances as iron, platinum, and oxygen are said to be *paramagnetic* or *magnetic.*

Many substances such as bismuth, antimony, zinc, lead, etc., when near a magnet are magnetized in such a way as to be *repelled* ‡ by the magnet. Such substances are said to be *diamagnetic.*

A rod of paramagnetic substance held parallel to the lines of force in a magnetic field is so magnetized by the field that its north pole points in the direction of the field, and the lines of force tend to converge into the rod and pass through it as explained in Art. 18.

* See *Philosophical Magazine* (5), Vol. 30, p. 205.

† A group of magnets mounted on pivots may be in equilibrium in a great variety of distinct configurations.

‡ Compare Art. 18, *Remark.*

A rod of diamagnetic substance held parallel to the lines of force in a magnetic field is so magnetized by the field that its south pole points in the direction of the field, and the lines of force tend to spread out and pass around the rod.

A rod of paramagnetic substance suspended in a magnetic field sets itself parallel to the field.

A rod of diamagnetic substance suspended in a magnetic field sets itself at right angles to the field.

Explanation.—Consider an iron rod in a magnetic field, f, as shown in Fig. 19. The component of the field parallel to the

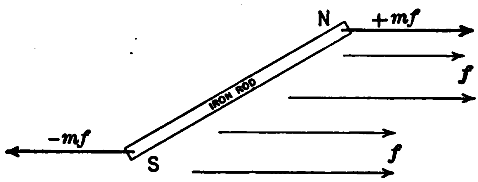


Fig. 19.

rod magnetizes the rod as indicated in the figure; the field pulls on the poles of the rod with the forces $+ mf$ and $- mf$ and tends to turn the rod parallel to f.

Consider a bismuth rod in a magnetic field, f, as shown in Fig. 20. The component of the field parallel to the rod magnetizes the rod as indicated in the figure; the field pulls on the poles of the rod with the forces $+ mf$ and $- mf$ and tends to turn the rod at right angles to f. If the bismuth rod, Fig. 20, is turned beyond the position at right angles to f, the magnetism of the rod is reversed, so that the field always pulls the rod towards the position at right angles to f.

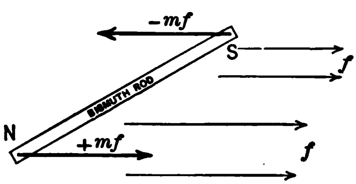


Fig. 20.

Weber's theory of diamagnetism.—A mass of copper near the end of an iron rod has electrical currents induced in it when the rod is suddenly magnetized, and so long as this current continues to flow in the copper the copper is strongly repelled by the magnet, and the lines of force tend to spread out from the copper and pass around it. The electrical resistance of the copper, however, very soon stops the induced current, when the strong repulsion ceases. The diamagnetic property of a substance has been explained by Weber upon the hypothesis that the molecules of the substance are perfect electrical conductors so that *permanent* currents are induced in the molecules when the substance is brought near a magnet.

25. Magnetic flux.—Let a (square centimeters) be an area at right angles to the velocity of a moving fluid, and let v (cm. per second) be the velocity of the fluid. Then av is the flux of fluid across the area in cubic centimeters per second. Similarly the product of the intensity, f, of a magnetic field into an area, a, at right angles to f is called the *magnetic flux* across the area. That is

$$\Phi = af \tag{9}$$

in which Φ is the magnetic flux across an area a which is at right angles to a magnetic field of intensity f.

Representation of the magnetic flux across an area, by the number of lines of force which pass through the area.—Imagine a surface drawn across a magnetic field so that the surface is at each point perpendicular to the magnetic field. Imagine lines of force drawn through the field so that the number of lines which pass through any square centimeter of this surface is equal to the intensity of the magnetic field at that part of the surface. Then *the magnetic flux passing through any area anywhere in the field is equal to the number of these lines of force that cross the area.** The unit flux (that is the flux across a square centimeter at right angles to a unit field) is therefore called the *line of force*, or simply the *line*, and a magnetic flux is specified as so many lines.

26. Magnetic flux from a magnet pole of strength m. *Proposition.*—The number of lines of force which emanate from a magnet pole of strength m is

$$\Phi = 4\pi m \tag{10}$$

Proof: Imagine a spherical surface of radius r drawn with the pole m at its center. The area of this spherical surface is $4\pi r^2$,

* The truth of this statement is by no means self evident. This matter is fully discussed in Chapter XVII.

the field intensity at this surface due to the pole is $\frac{m}{r^2}$ according to equation (3), and this field is everywhere at right angles to the spherical surface. Therefore, according to equation (9), the magnetic flux across the spherical surface is $\frac{m}{r^2} \times 4\pi r^2 = 4\pi m$. Q. E. D.

CHAPTER II.

THE ELECTRIC CURRENT.

27. Preliminary statements.—The production of an electric current requires a *generator*, such as a battery (§ 72) or a dynamo (§ 57). The path of the current is termed an *electric circuit.* When the path is complete, leading out from the generator and returning to it without break or interruption, the circuit is said to be *closed.* A steady electric current always flows in a *closed circuit*, that is, a circuit which goes out from the generator and returns to it.

The generator of an electric current must always be supplied with energy in some form, and a portion of this energy reappears in the various parts of the circuit through which the current flows. Thus, energy reappears as heat in an electric lamp and as mechanical work in an electric motor.

The most important effects of the electric current are the magnetic effect, the heating effect, the chemical effect, and the physiological effect. The first three effects are capable of accurate measurement and they constitute therefore the chief basis of the mathematical theory of the electric current.

The magnetic effect.—A wire through which an electric current is flowing is called an *electric wire* for brevity. The region surrounding an electric wire is a magnetic field (see § 28), and it is to the action of this field that many of the most interesting phenomena of the electric current are due.

The heating effect.—A wire, or any substance which forms a portion of an electric circuit, has heat generated in it by the current. This heating effect of the electric current is exemplified in the ordinary glow lamp, the carbon filament of which forms a portion of an electric circuit and is heated to incandescence.

The chemical effect.—When a solution of a chemical compound forms a portion of an electric circuit the compound is in general decomposed by the current. This chemical effect of the electric current is exemplified in the practical operation of electroplating.

The physiological effect.—When the human body forms a portion of an electric circuit certain effects are produced which depend mainly upon the excitation of the nerves which lie in the path of the current. Sensations of taste, of smell, of sight or of hearing are produced when the current passes through the regions occupied by the nerves of taste, of smell, of sight or of hearing, respectively, and the muscles through which the current passes are more or less violently contracted.

The hydraulic analogue of the electric current.—The electric current in a wire may conveniently be looked upon as something flowing through the wire very much as water flows through a pipe. Thus a battery or a dynamo producing an electric current in a circuit of wire is analogous to a pump forcing water through a pipe, the pipe leading out from the pump and returning to it, so that the water is pumped through the pipe again and again. Energy must be supplied to the pump, as to an electric generator, and this energy reappears in the various parts of the circuit through which the water flows. Thus heat is generated because of the frictional resistance of the pipe, and if a water motor is included in the water circuit a portion of the energy expended at the pump reappears as mechanical work in the motor.

Remark.—Water flowing through a pipe produces in the region surrounding the pipe no disturbance corresponding to the magnetic field surrounding an electric wire. Therefore *the hydraulic analogue of the electric current is of no help in giving one a conception of the magnetic effect of the electric current,* and in the study of those phenomena of the electric current which depend upon its magnetic effect the hydraulic analogue must be used with caution.

28. The magnetic field due to an electric wire.—The lines of force of the magnetic field produced by an electric wire encircle the wire. A north magnet pole tends to move round the wire in one direction and a south magnet pole tends to move round the

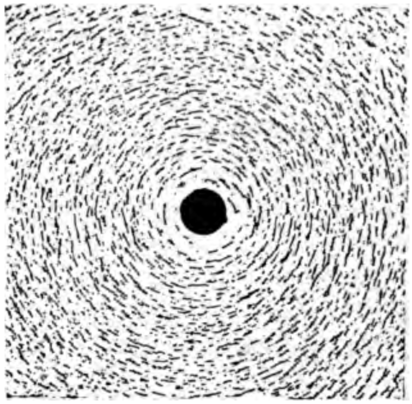


Fig. 21.

wire in the opposite direction. Fig. 21 shows the magnetic lines of force encircling the straight portion of an electric wire. This

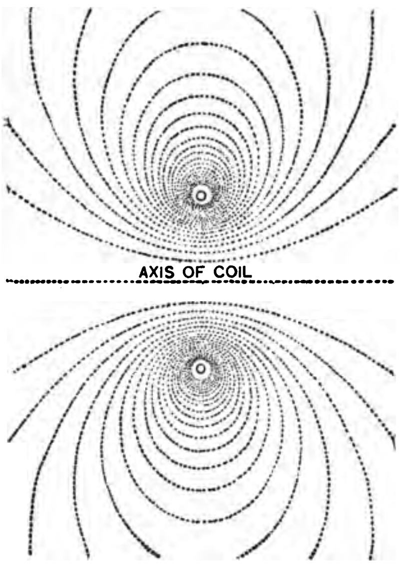


Fig. 22.

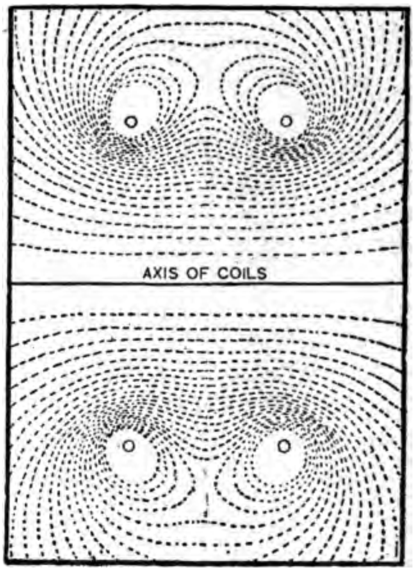


Fig. 23.

figure is a photograph of a magnetic figure obtained by passing the wire vertically through a small hole in a horizontal glass plate upon which iron filings are dusted.

Fig. 22 shows the trend of the magnetic lines of force in the neighborhood of a circular loup of wire. The plane of the circular loup is at right angles to the plane of the figure, and the two very small circles represent the section of the wire where it passes through the plane of the figure.

Fig. 23 shows the trend of the magnetic lines of force in the neighborhood of two circular loups of wire side by side. The planes of the loups of wire are perpendicular to the plane of the figure, the heavy line is the common axis of the two loups of wire, and the four small circles represent the section of the wire where it passes through the plane of the figure.

29. The composite magnetic field produced when a straight electric wire is stretched across a magnetic field which, but for the presence of the electric wire, would be a uniform field.—The magnetic field between the flat-ended magnet poles, Fig. 16, is

Fig. 24.

sensibly uniform. Suppose an electric wire be stretched between the poles perpendicular to the plane of the paper. The composite field due to magnet poles and electric wire together is shown in Fig. 24 which is from a photograph. The trend of the lines of force is more clearly shown in Fig. 25 which shows the theoretical trend of the lines of force near an electric wire stretched across a magnetic field which would be *uniform* were it not for the disturbing influence of the electric wire.

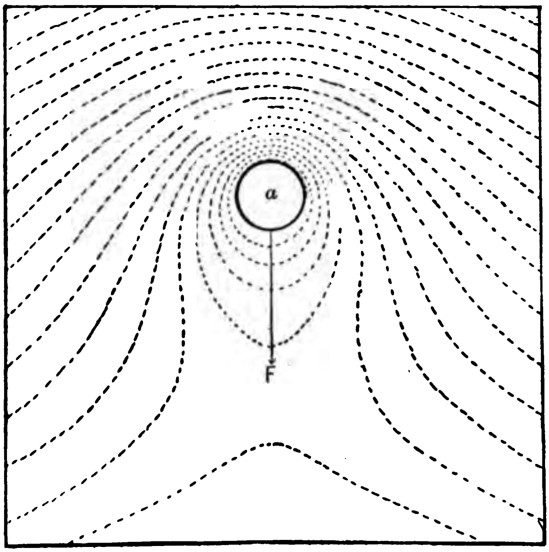


Fig. 25.

Side push on an electric wire stretched across a uniform magnetic field.—The electric wire *a*, Figs. 24 and 25, is pushed sidewise by the magnetic field as indicated by the arrow *F*. This side force *F* is at right angles both to the magnetic field and to the wire. This side force may be ascribed to the tendency of the lines of force to shorten. This side push on an electric wire in a magnetic field is exemplified in the electric motor. The electric current which is supplied to the motor passes through a number of wires which are pushed sidewise across the face of a magnet pole.

30. Strength of current.—Consider a straight electric wire stretched across a uniform magnetic field of unit intensity, the wire being at right angles to the field as described in the foregoing article. *The force in dynes with which the field pushes sidewise on one centimeter of this wire is used as the numerical measure of the strength of the current in the wire.* This force-per-unit-length-of-wire-per-unit-field-intensity is hereafter called, simply, the strength of the current in the wire, let it be represented by i. The force pushing sidewise on l centimeters of the wire is il and if the field is f units intensity, instead of one unit intensity, the force is f times as great or ilf. Therefore:

$$F = ilf \tag{11}$$

in which F is the force in dynes pushing sidewise upon l centimeters of wire at right angles to a uniform magnetic field of intensity f, and i is the strength of the current in the wire. When the force F is expressed in dynes, the length l in centimeters and the field f in c.g.s. units, then the current strength i in equation (11) is expressed in c.g.s. units of current.

The c.g.s. unit of current is a current of such strength that one centimeter of the wire in which it flows is pushed sidewise with a force of one dyne when the wire is at right angles to a magnetic field of unit intensity.

The ampere, which is the practical unit of current, is defined as *one-tenth* of the c.g.s. unit of current.

Direction of current.—A magnet pole tends to move round an electric wire as stated in article 28. *The current is said to flow along a wire in the direction in which a right-handed screw (coaxial with the wire) would move if turned in the direction in which a north magnet pole tends to move round the wire.*

Remark.—The relative direction of current in a wire, magnetic field f in which the wire is placed, and side push F is as follows: The field due to the wire circles round the wire in the direction in which a right-handed screw would have to be turned to make the screw move in the direction of the current. This field due to the wire works against or weakens the field f on one side of the wire, and strengthens it on the other side, as shown in Fig. 25. The wire is pushed away from the side upon which the composite field is intense.

Kirchhoff's law.—The strength of a steady electric current is the same in all parts of a circuit. That is, the different parts of a circuit are acted upon by the same amount of side push per centimeter of length when held at a given distance from a given magnet pole. This fact, which was first clearly stated by Kirchhoff, is indirectly confirmed by a great variety of experimental results. A simpler statement of Kirchhoff's law is given in article 51. The hydraulic analogue of the electric current gives a clear idea of Kirchhoff's law. The electric current in a wire is analogous to the flow of an *incompressible* fluid in a pipe and the same volume of fluid per second passes through each portion of the pipe.

Force action on an electric wire which is not at right angles to a magnetic field.—When an electric wire is parallel to a magnetic field no force acts on the wire. If the angle between the wire and the direction of the field is θ then the field may be resolved into the components $f \sin \theta$ and $f \cos \theta$ perpendicular to and parallel to the wire respectively. The latter component has no action on the wire, while the former component produces the side force

$$F = ilf \sin \theta \tag{12}$$

If the wire is not straight or the field not uniform then attention must be directed to an *element* of the wire and equation (12) becomes:

$$\Delta F = if \sin \theta \cdot \Delta l \tag{13}$$

in which Δl is a short portion, or element, of the wire, f is the intensity of the field at the element, θ is the angle between f and Δl, i is the strength of current in the wire, and ΔF is the force pushing on Δl. This force is perpendicular both to f and to Δl.

31. Contribution to the magnetic field at a given point by one element of an electric wire.—The region surrounding an electric wire is a magnetic field, and each element of the wire may be considered as contributing its share to the field intensity at each point. Let p, Fig. 26, be the point at which it is desired to find the field intensity Δf produced by a given element Δl of the electric wire. Let r be the distance from p to Δl, and let θ be the angle between r and Δl as shown in Fig. 26. Let a magnet pole of strength m be placed at p. The field intensity *at the element* Δl due to this pole is $\frac{m}{r^2}$ according to equation (3). This field, according to equation (13), pushes on Δl with the force

$$\Delta F = i \frac{m}{r^2} \sin \theta \cdot \Delta l$$

This is the force with which the pole m acts on Δl and therefore it is also the force (disregarding sign) with which Δl reacts upon m. But the force with which the element Δl acts upon m is equal to the field intensity at m, due to Δl, multiplied by m. That is $\Delta F = m \cdot \Delta f$ or

$$\Delta f = \frac{i \sin \theta}{r^2} \cdot \Delta l \qquad (14)$$

in which Δf is the field intensity at p (Fig. 26) due to the element Δl. This field Δf at p is perpendicular to r and to Δl.

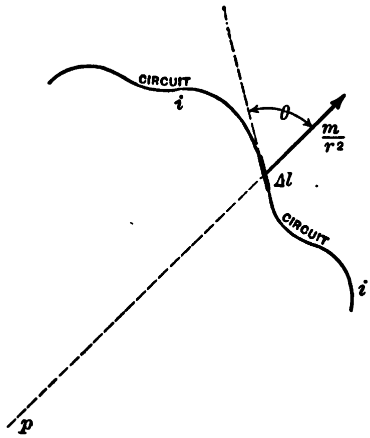


Fig. 26.

32. Total intensity at a point of the field due to an entire electric circuit.—The total field intensity, f, at a point is the sum (vector sum) of the contributions by the various elements of a circuit. Therefore from equation (14) we have

$$f = i\Sigma \frac{\sin \theta}{r^2} \cdot \Delta l \qquad (15)$$

This summation, being a vector summation, requires some further explanation. From the point p (Fig. 26), draw a *line* representing the value of $\Delta f = \frac{i \sin \theta \Delta l}{r^2}$ at that point due to a given element Δl of the current. The direction of Δf will be perpendicular to the plane of r and Δl. From the terminus of this line, draw another representing, in the same manner, the contribution to the field at p due to the next element of the current, and so on for the whole circuit. The line drawn from p to the point finally so reached will then represent the intensity of field at p due to the whole circuit.

Remark.—The intensity of the magnetic field, at any given point in the neighborhood of a given coil of wire, is proportional to the current i in the coil, and the direction of the field does not vary with the strength of the current. Therefore the field intensity f at a *given point* in the neighborhood of a coil of wire is

$$f = Gi \qquad (16)$$

in which G is a constant for the given point and i is the strength of the current flowing in the coil.

33. The intensity of the field at the center of a circular loup of wire.—In this case the factor G, equation (16), is easily calculated from the dimensions of the loup and the number of turns of wire. A magnet pole m placed at the center of the loup as shown in Fig. 27 produces *at the wire* a field of intensity $\frac{m}{r^2}$, r being the

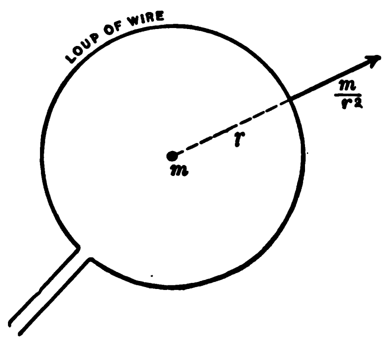


Fig. 27.

radius of the loup. This field is everywhere at right angles to the wire and in the plane of the loup. The length of the wire is $2\pi rn$, where n is the number of turns of wire in the loup, so that $i \times 2\pi rn \times \frac{m}{r^2}$ is the force with which the wire is pushed sidewise by the pole m. This is also the force with which the loup pushes the pole in the opposite direction. Now the force acting on the pole is fm where f is the field intensity at the pole due to the loup.

Therefore

$$f = \frac{2\pi ni}{r} \qquad (17)$$

The field intensity at a point in the axis of a circular loup of ire of radius r, the point being at a distance d from the plane of loup is

$$f = \frac{2\pi n r^2 i}{(r^2 + d^2)^{\frac{3}{2}}} \tag{18}$$

Proof of equation (18).—Imagine a pole m placed at the given point. The field intensity at the wire due to this pole is $\frac{m}{r^2 + d^2}$. The component of this field in the plane of the loup is $\frac{m}{r^2 + d^2} \times \frac{r}{\sqrt{r^2 + d^2}}$ and this component pushes the wire sidewise with a force equal to $i \times 2\pi r n \times \frac{rm}{(r^2 + d^2)^{\frac{3}{2}}}$. This same force reacts on the pole and is equal to mf so that $f = \frac{2\pi n r^2 i}{\sqrt{r^2 + d^2}}$. Q. E. D.

Remark.—A winding of wire is called a *coil.* The usual form of coil consists of wire wound upon a spool. When the spool is very short, and large in diameter the coil is called a *circular coil.* When the spool is long in comparison with its diameter the coil is called a *solenoid.* The solenoid is often made by winding wire in one or more layers upon a long tube.

34. The tangent galvanometer is an instrument for measuring an electric current. It consists essentially of a circular coil of

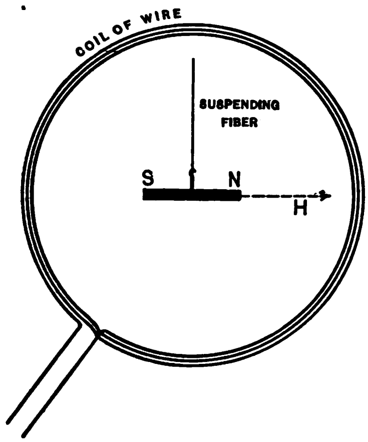


Fig. 28.

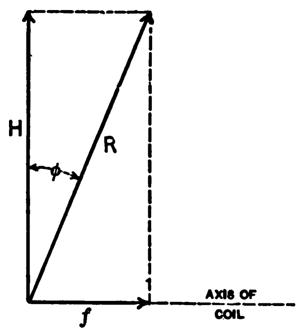


Fig. 29.

wire, at the center of which a small magnet is suspended, as shown in Fig. 28. This suspended magnet carries a pointer which plays over a divided circle so that one may observe the angle through which the magnet is turned when a current is sent through the wire. The coil of wire is mounted with its plane vertical and in the direction of the horizontal component, H, of the earth's magnetic field.

When no current flows through the coil the suspended magnet points in the direction of H. A current i in the coil produces at the center of the coil a magnetic field, $f = \frac{2\pi ni}{r}$, at right angles to H. This field f compounded with H gives a resultant field R, Fig. 29, in the direction of which the suspended magnet now points. From Fig. 29 we have

$$\tan \phi = \frac{f}{H} \tag{19}$$

Writing for f its value $\frac{2\pi ni}{r}$, and solving for i we have

$$i = \frac{rH}{2\pi n} \cdot \tan \phi \tag{20}$$

This equation gives i when r, H, and n are known and ϕ observed. If i is to be expressed in amperes then

$$i_{\text{amp.}} = \frac{5rH}{\pi n} \cdot \tan \phi \tag{21}$$

for the number which expresses a current in amperes is ten times as large as the number which expresses the same current in c.g.s. units.

Reduction factor.—The quantity $\frac{rH}{2\pi n}$ $\left(\text{or } \frac{5rH}{\pi n}\right)$ by which the tangent of the observed deflection is multiplied to give the value of the current is called the *reduction factor* of the galvanometer. Representing this factor by k we have

$$i = k \tan \phi \tag{22}$$

The Helmholtz-Gaugain type of tangent galvanometer.—Equations (20) and (21) assume that the suspended magnet is *exactly at the center* of the circular coil of wire and that the magnet is *very short*, so that it will point in the direction of the resultant field at that point. These two conditions cannot, of course, be exactly realized in practice. Fig. 23 shows that between two similar circular coils side by side there is an extended region throughout which the magnetic field due to the coils is sensibly uniform, and Helmholtz and Gaugain have pointed out, that, by using such a pair of coils with a magnet suspended midway between them as a tangent galvanometer, no perceptible error is introduced by inaccurate centering of the suspended magnet, nor is any appreciable error produced by using a comparatively long magnet.

35. The comparison of currents by means of the tangent galvanometer.—The use of the tangent galvanometer for the determination of current values requires a knowledge of the reduction factor, k, of the galvanometer. This reduction factor depends upon the intensity of the earth's horizontal field H, which is more or less variable and therefore seldom accurately known.

The tangent galvanometer may be used to determine the ratio of two currents when the reduction factor is not known. For this purpose a current i' is sent through the galvanometer and the deflection ϕ' is observed. Then

$$i' = k \tan \phi' \qquad (a)$$

Another current i'' is then sent through the instrument and the deflection ϕ'' is observed. Then

$$i'' = k \tan \phi'' \qquad (b)$$

Dividing equation (a) by equation (b) member by member we have

$$\frac{i'}{i''} = \frac{\tan \phi'}{\tan \phi''}$$

in which the factor k does not appear.

36. The action of a uniform magnetic field upon a suspended coil in which an electric current is flowing.—A coil of wire suspended in a magnetic field and supplied with an electric current tends to place itself so that the greatest possible number of lines of force may pass through the opening of the coil, and so that the magnetic field inside of the coil due to the current may be in the same direction as the field which is acting on the coil. If the magnetic field is not uniform the forces which tend to bring the coil into this position may tend to produce both translatory motion and rotatory motion of the coil. If the magnetic field is uniform the forces which act on the coil tend to produce rotatory motion only. The simplest case is that of a rectangular coil with its plane lying in the direction of the field.

Consider a rectangular coil *CC*, Fig. 30, suspended so as to be free to rotate about the axis *pp*, in a uniform magnetic field of

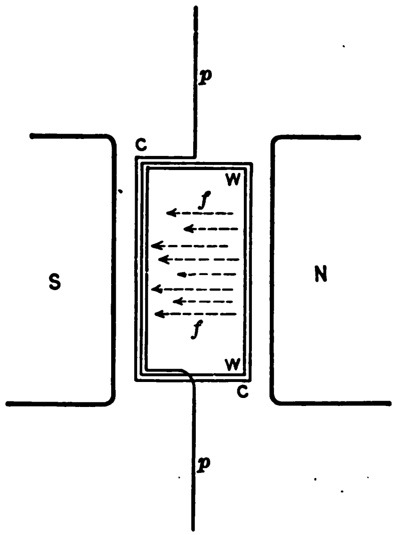


Fig. 30.

intensity f as shown. Let b be the breadth of the coil, a its length parallel to *pp*, n the number of turns of wire in the coil, and i the current flowing in the coil. The wires *ww* in one side of the coil have a total length na and according to equation 11 are pushed sidewise by the field with a force $i \cdot na \cdot f$. The torque action of this force about the axis *pp* is $b/2 \cdot i \cdot na \cdot f$. The wires which form the other side of the coil are pushed with a force which also produces a torque $b/2 \cdot i \cdot na \cdot f$ so that the total torque T, tending to turn the coil about the axis *pp* is

$$T = abnif \tag{23}$$

The torque acting upon a circular coil of radius r is:

$$T = \pi r^2 nif \tag{24}$$

When the plane of the coil makes an angle θ with the direction of the field f, the component of the field parallel to the plane of the coil is $f \cos \theta$ which, written in place of f in equations (23) and (24), gives:

$$T = abnif \cos \theta \tag{25}$$

and

$$T = \pi r^2 nif \cos \theta \tag{26}$$

Remark.—In general the torque tending to turn a coil of wire which is suspended in a uniform magnetic field is $Aif \cos \theta$, where A is the sum of the areas enclosed by the respective turns of wire, i is the current in the coil, f is the intensity of the uniform magnetic field and θ is the angle between f and the plane of the coil.

37. The electrodynamometer is an instrument for measuring the strength of an electric current by means of the mutual force action between two coils of wire through both of which the current to be measured is sent. One of these coils is fixed and the other is suspended so as to move freely. The magnetic field produced by the fixed coil exerts a force upon the movable coil and this force, or the movement which it produces, is observed.

The absolute electrodynamometer is so constructed that the strength of a current may be calculated from the observed force action together with the measured dimensions of the instrument. The simplest absolute electrodynamometer is that devised by Wilhelm Weber in 1846. It consists of a large circular coil rigidly mounted with its plane vertical, and a small circular coil suspended at the center of the large coil by two fine wires. A current i is sent through both coils. The field produced by the outer coil at its center is $f\left(=\frac{2\pi n'i}{r'}\right)$, where n' is the number of turns of wire in the coil and r' is its radius. This field exerts a torque $T(=\pi r''^2 n'' if \cos \theta)$ upon the small coil, where n'' is the number of turns of wire in the small coil, r'' is its radius, and θ is the angle between f and the plane of the small coil. Substituting the value $f = \frac{2\pi n'i}{r'}$ in the expression for T we have

$$T = \frac{2\pi^2 n' n'' r''^2 i^2}{r'} \cos\theta \qquad (27)$$

This equation permits the calculation of i when n', n'', r' and r'' are known and T and θ have been observed. Fig. 31 shows a later and slightly modified form of Weber's absolute electrodynamometer, in which the suspended coil is in the approximately uniform field between two large circular coils.

Fig. 31.

The Siemens type of electrodynamometer.—The force action between two coils is proportional strictly to the square of the current which flows through the two coils, whatever the shape and tive position of the two coils may be, provided only that the tive position of the two coils does not change. Therefore if force action between the coils is measured first for a current id then for a current i'', the ratio i'/i'' is equal to the square of the ratio of the observed force actions. The electro-namometer designed by Siemens is used for measuring current itios in this way. The coils are both rather large and they are placed near together so that the force action may be great enough to be easily measured. The coil *A*, Fig. 32, is held stationary by the frame of the instrument, and the coil *B* is suspended by a fine silk thread and hangs with its plane at right angles to the plane of the coil *A*. The coil *B* is sometimes a single turn of wire as shown in the diagram, Fig. 33. The movable coil is provided with flexible or mercury cup connections

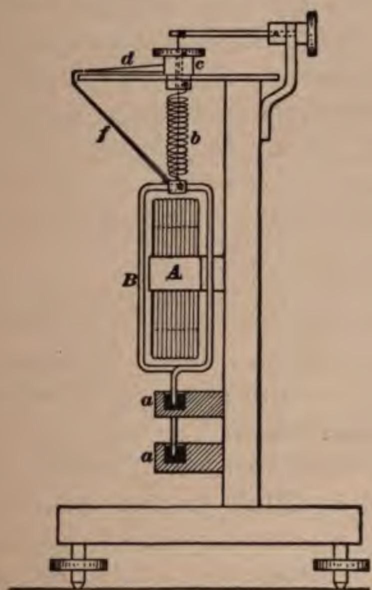


Fig. 32.

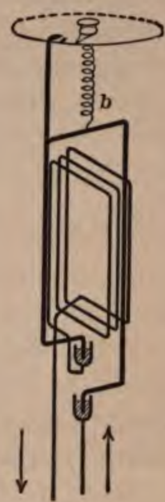


Fig. 33.

a a and the current to be measured is sent through both coils in series. The force action between the coils is balanced by carefully twisting a helical spring *b*, one end of which is attached to coil *B* and the other to the torsion head *c*. The observed angle of twist necessary to bring the swinging coil to its standard position is read off by means of the pointer *d* and a graduated circle. The pointer *f* attached to the movable coil shows when the latter has been brought to its standard position. The observed angle of twist of the helical spring affords a measure of the force action between the coils, and the current is proportional to the square root of this angle.

39. The sensitive galvanometer (Kelvin).—From the equation (20) of the tangent galvanometer it is evident that a given current will produce the greatest deflection ϕ when the number of turns of wire in the coil is very great, when the radius of the coil is very small, and when the directing field H is weak. A galvanometer constructed so as to fulfil these three conditions is called a *sensitive* galvanometer. Such a galvanometer is used principally for the detection of electric current.

The magnet of such an instrument is suspended by means of a fine fiber of unspun silk or quartz. In order that small deflections may be easily detected it is customary to attach a small mirror to the suspended magnet, and to observe with a telescope and scale.

Use of governing magnets.—In order to secure a weak directing field H, the earth's field is usually partially neutralized in the neighborhood of the suspended magnet needle by superposing upon it an opposing field due to a large magnet rightly placed in the neighborhood of the galvanometer. This magnet is called a *governing* magnet. This device, however, introduces certain difficulties. The earth's field is slightly variable, while the opposing field, due to a governing magnet, is constant. The fluctuations in H become very troublesome, therefore, when the attempt is made to gain extreme sensitiveness by the use of a governing magnet.

Fig. 34.

Use of an astatic system of magnets.—A method for further increasing the sensitiveness of galvanometers consists in the use of what is termed an astatic system of magnets.

Two magnets, NS and SN, of equal magnetic moments, attached to a rod, as shown in Fig. 34, constitute an astatic system. Such a system if suspended in the earth's field will point indifferently in any direction. If one magnet is slightly stronger the other, or if their axes do not lie in the same plane, the earth's field will exert only a very slight directing action upon the system. Such a system may be suspended with one of its magnets inside of a galvanometer coil as shown in Fig. 35; or two coils, so connected that a current sent through the instrument flows in opposite directions in them, may be used, one surrounding each magnet, as shown in Fig. 36. This latter design is due to Lord Kelvin. A galvanometer so constructed, with very short magnets, light connecting rod and mirror, and coils containing many turns of wire, can be made to *indicate*, distinctly, currents which do not exceed 10^{-12} amperes.

Fig. 35.

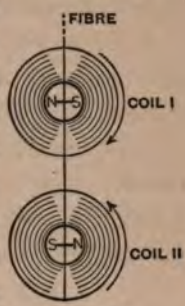


Fig. 36.

The Kelvin galvanometer may be used for the *approximate measurement* of weak currents since the deflection, within a small range, is proportional to the current.

39. The sensitive galvanometer (D'Arsonval).—A coil suspended in a magnetic field is, as has been shown (§ 36), acted upon by the torque

$$T = abnif$$

If the coil is suspended by wires this torque will produce a slight rotational movement of the coil. In order that the coil may be perceptibly moved by a very weak current the suspending wires, which also serve to lead current to and from the coil, must be very fine, the number of turns of wire in the coil must be very great and the field f in which the coil is suspended must be very intense. To obtain a quick movement of the coil it is important to have its lateral dimensions small. Fig. 37 shows the essential parts of a sensitive galvanometer constructed according to these prin-

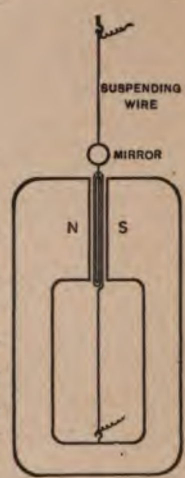


Fig. 37.

ciples. It consists of an elongated coil of fine wire suspended in the strong field between the poles of a horseshoe magnet. This type of galvanometer is due to D'Arsonval. It is not so sensitive as the Kelvin type of galvanometer but it is scarcely at all affected by outside magnetic influences; it can, indeed, be used in the same room with any kind of electrical machinery. The D'Arsonval galvanometer may be used for the approximate measurement of weak currents inasmuch as the deflection within a small range is sensibly proportional to the current. The deflections are usually read by means of a telescope and scale.

CHAPTER III.

RESISTANCE AND ELECTROMOTIVE FORCE.

Resistance.

40. Electrical resistance. Conductors and insulators.—When a pump forces water through a circuit of pipe a part of the work expended in driving the pump reappears as heat in the various parts of the circuit of pipe because of the resistance which the pipe offers to the flow of water.

Similarly, when an electric generator produces an electric current in a circuit, a part of the work expended in driving the generator reappears as heat in the various parts of the circuit. The electric current seems to be opposed by a kind of *resistance* more or less analogous to the resistance which a pipe offers to the flow of water. A portion of a circuit is said to have more or less *electrical resistance* according as more or less heat is generated in it by a given current.

A substance which has comparatively low electrical resistance is called a *conductor*. The metals, carbon, and salt solutions are good conductors.

A substance which has very great electrical resistance is called an *insulator*. Air, glass, and hard rubber are good insulators. By using a very sensitive galvanometer it may be shown that a perceptible current can be made to flow through any substance.

A wire, or any conductor, is said to be *insulated* when it is separated by insulating substances from all neighboring conductors. Thus telegraph and telephone wires are insulated by being supported upon glass or porcelain knobs. Wire for use in winding electromagnets is insulated by a covering of cotton or silk.

41. Joule's law. Measure of resistance.—The rate at which heat is generated in a given wire is proportional to the square of the current flowing in the wire. That is

$$\dot{H} = Ri^2 \tag{28}$$

in which $\dot{H}$ is the rate of generation of heat (units of heat per second) in the wire, i is the strength of the current flowing in the wire, and R is the proportionality factor. This quantity R has a definite value for a given wire and it is adopted as the numerical measure of the resistance of the wire.

If the current i is constant then the total amount of heat, H, generated in the wire in t seconds is $\dot{H}t$ and from equation (28) we have

$$H = Ri^2t \tag{29}$$

The c.g.s. unit of resistance.—If H in equation (29) is expressed in ergs * and i in c.g.s. units of current, then R is expressed in terms of a unit called the c.g.s. unit of resistance; that is, a wire has one c.g.s. unit of resistance when one erg of heat is generated in it in one second by one c.g.s. unit of current.

The ohm.—If H in equation (29) is expressed in joules (1 joule $= 10^7$ ergs), and i in amperes, then R is expressed in terms of a unit called the *ohm*; that is, a wire has one ohm of resistance when one joule of heat is generated in it in one second by one ampere of current. The ohm is equal to 10^9 c.g.s. units of resistance.

42. Power required to maintain a current in a circuit, expressed in terms of resistance and current.—When the energy which reappears in an electric circuit reappears as *heat only* then the rate at which work is expended in maintaining the current is equal to the rate at which energy reappears in the circuit as heat. In a circuit of resistance R energy reappears as heat, according to

* The erg is the c.g.s. unit of energy or work and heat may therefore be expressed in ergs.

equation (28), at the rate Ri^2. Therefore the power P required to maintain a current i in a circuit of resistance R is:

$$P = Ri^2 \qquad (30)$$

If R and i in this equation are expressed in c.g.s. units then P is expressed in ergs per second. If R is expressed in ohms and i in amperes then P is expressed in joules per second, or in watts.

43. Specific resistance.—The resistance R of a wire of a given material is found to be directly proportional to its length l, and inversely proportional to its sectional area q; that is,

$$R = k\frac{l}{q} \qquad (31)$$

The proportionality factor k is called the *specific resistance* of the material. It is equal to the resistance of a wire of unit length ($l = 1$) and unit sectional area ($q = 1$).

The accompanying table gives the value of k (resistance in ohms of a wire one centimeter long and one square centimeter sectional area) for various substances.

TABLE OF SPECIFIC RESISTANCE (RESISTIVITY).

METAL.	RESISTANCE OF A BAR 1 CM. LONG, 1 SQ. CM. CROSS-SECTION AT 0° C.
Aluminium (annealed)	0.00000289 ohm.
Copper (annealed)	0.00000160
Gold	0.00000208
Iron (pure)	0.00000964
Iron (telegraph wire)	0.00001500
Lead	0.00001963
Mercury	0.00009434
Platinum	0.00000898
Silver (annealed)	0.00000149
German Silver (Cu 60, Zn 26, Ni 14)	0.000021 ohm.
Platinoid (Cu 59, Zn 25.5, Ni 14, W 55)	0.000032
Manganin (Cu 84, Ni 12, Mn 3.5)	0.000047

Remark.—The reciprocal of the resistance of a circuit is called its *conductivity*. The conductivity of a wire is directly proportional to its sectional area and inversely proportional to its length. The proportionality factor is called the *specific conductivity* of the material. Specific conductivity is the reciprocal of specific resistance.

44. Influence of temperature upon resistance.—The resistance of a wire depends not only upon the material of which the wire is made, but also upon the temperature of the material.

The increase in the resistance of a given wire, due to a rise in temperature, is proportional to the initial resistance and approximately proportional to the rise in temperature; that is, if R_0 is the resistance of a wire at some standard temperature, say at zero centigrade, the increase of resistance when the wire is warmed to $t°$ C. is $\beta R_0 t$, where β is the proportionality factor. Therefore the total resistance of the wire at $t°$ C. is $R_t = R_0 + \beta R_0 t$, or,

$$R_t = R_0(1 + \beta t) \tag{32}$$

The quantity β is called the *temperature coefficient of resistance of the given material.* For many pure metals β has nearly the same value, viz., 0.0037; that is, the resistance of a pure metal is very nearly proportional to the absolute temperature, as measured by an air thermometer. The value of β for pure commercial copper is about 0.0040.

Salt and acid solutions and graphitic carbon diminish in resistance with rise of temperature, so that these substances have negative temperature coefficients of resistance.

The influence of temperature upon resistance has been thoroughly investigated between − 200° C. and + 200° C. Figure 38 gives a graphic representation of the results obtained by Dewar and Fleming.* The ordinates of the curves are specific resistances; they are expressed in c.g.s. units, and to be reduced to ohms they must be divided by 10^9.

* Dewar and Fleming, Philosophical Magazine (5), Vol. 34, p. 326; Vol. 36, p. 271. Also Price, Measurement of Electrical Resistance, p. 17.

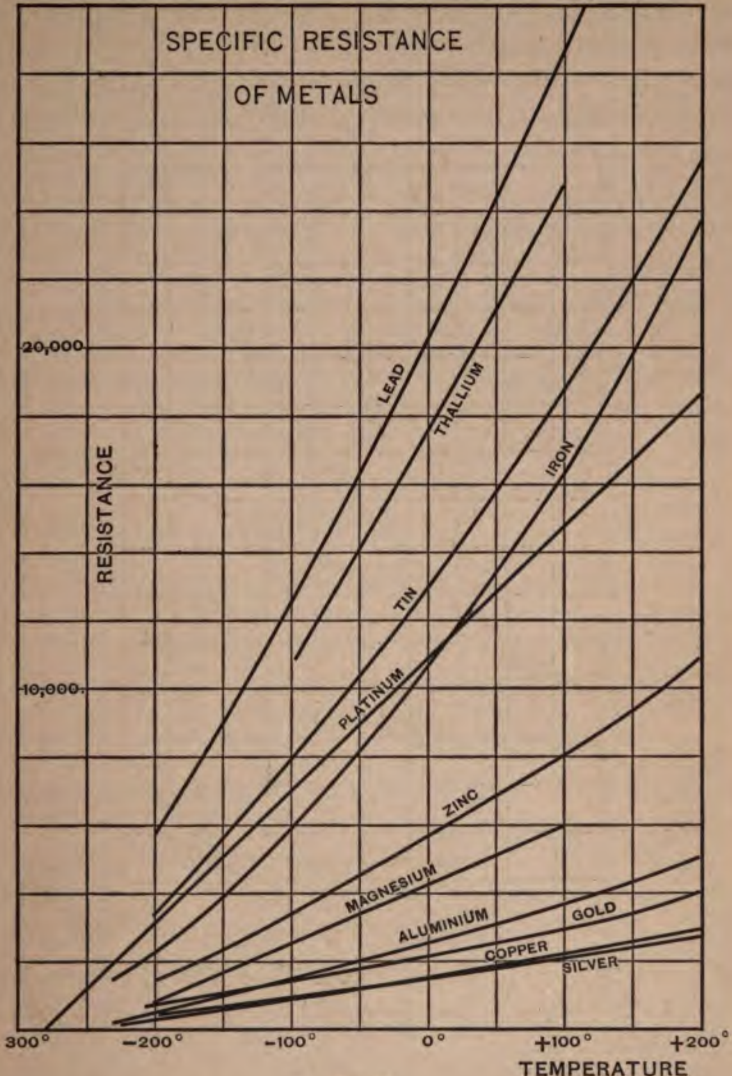


Fig. 38.

45. Specific resistance and temperature coefficients of alloys.— The ordinates of the three curves, Fig. 39,* represent the specific resistance at a given temperature of alloys of zinc and tin, of

* From the results of Matthiesen, Philosophical Transactions, 1862.

silver and gold, and of silver and platinum respectively, and the abscissas represent percentages of the constituent metals. The zinc-tin line, marked *Zn + Sn*, is sensibly straight; that is, the change of resistance from pure zinc to pure tin is proportional to

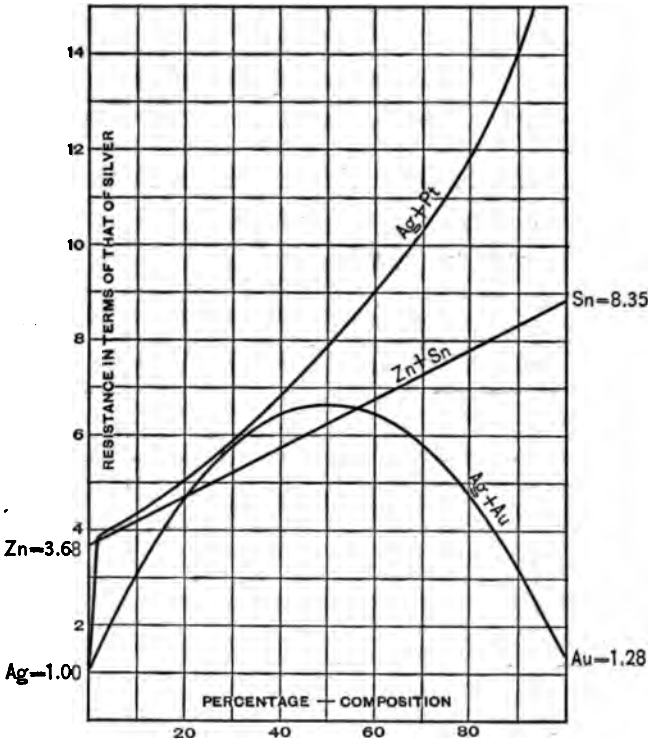


Fig. 39.

the percentage of tin in the alloy. The silver-platinum line, marked *Ag + Pt*, and the silver-gold line marked *Ag + Au* are not straight.

In respect to electrical resistance the alloys of tin, lead, cadmium and zinc are similar to the alloys of zinc and tin. Alloys of most other metals are more or less similar to the alloys of silver and gold, and of silver and platinum. The addition of one of these metals, even in small quantity, to any other metal increases the specific resistance considerably and diminishes the temperature coefficient. The resistance of alloys, as a rule, changes less with temperature than do the resistances of pure metals.

Wire which is used for standards of resistance should not change its resistance with time. Such change of resistance is largely avoided by thorough annealing. It is also desirable that a resistance standard have a very low temperature coefficient so that the troublesome temperature corrections may be small in value. German silver, an alloy of copper, zinc, and nickel, has a temperature coefficient ranging from 0.00036 to 0.00044 according to the composition. An alloy, *manganin*, composed of 12 parts of nickel, 84 parts of copper and 4 parts of manganese, has a temperature coefficient which is very nearly zero. Thus a sample of this alloy having a resistance of 100 ohms at 10° C. has a resistance of 100.02 ohms at 20° C.

46. Measurement of resistance.—The resistance of a wire may be determined by measuring the heat H generated in the wire by a known current i in an observed interval of time t. The resistance of the wire may then be calculated from equation (29). This is the fundamental method for measuring resistance. It can not, however, be carried out with great precision on account of the difficulty of measuring heat accurately.

Indirect methods for accurately measuring resistance have been devised by Weber, Lorentz, and others.* Standard resistances have been determined with great care by these methods, and all ordinary laboratory methods for determining resistance are by comparing the resistance to be determined with a standard.

Electromotive Force.

47. Power delivered by an electric generator. Definition of electromotive force.—An electric generator such as a dynamo or a battery is analogous to a pump which develops a *certain definite difference of pressure* between its inlet and outlet. The or-

* See A. Gray, Absolute Measurements in Electricity and Magnetism, Vol. II., pp. 538–600.

dinary pump does not do this. The centrifugal pump, which is simply a rotary fan, does do this, and in order to gain a clear idea of the conditions which determine the amount of current and the amount of power delivered by an electric generator it is important to understand the action of the fan blower. Imagine a fan blower, driven at constant speed, and connected to a circuit of pipe through which a current of air is maintained by the blower, the air being returned to the blower. The blower maintains a constant pressure-difference between its outlet and inlet and the quantity of air per second which is forced through the circuit of pipe depends upon the resistance which the pipe offers to the air stream. If the pipe is long and small the air current (units volume of air per second) will be small. If the pipe is short and large a much larger quantity of air will pass per second. Let i be the volume of air per second delivered by the fan and let E be the pressure-difference between outlet and inlet of fan. Then the power delivered by the fan to the circuit of pipe is

$$P = Ei$$

This equation shows that if E is constant the power delivered by the fan is proportional to i.

Furthermore, if E' is the pressure-difference between any two points of the pipe circuit, then the power P' which is expended in the portion of the pipe which lies between these points is

$$P' = E'i$$

Remark.—The power delivered by a fan is not strictly proportional to the quantity of air delivered per second inasmuch as an increased flow of air means increased friction of the air in passing through the fan and consequent lowering of the pressure-difference developed by the fan.

The rate at which an electric generator delivers energy, or the power delivered by a given generator, is proportional to the current. That is:

$$P = Ei \tag{33a}$$

in which P is the power delivered by a generator to a circuit in which it produces a current i. The factor E is called the *electromotive force* of the generator.

Example.—When a dynamo produces current in a long, fine wire the current is small and but little power is delivered to the circuit. When the dynamo produces current in a shorter or larger wire the current is larger and more power is delivered by the dynamo.

Remark 1.—The electromotive force of an electric generator is analogous to the pressure-difference developed by a pump.

Remark 2.—The power delivered by an electric generator is not always strictly proportional to the current i inasmuch as an increase of current usually lowers the electromotive force of the generator, very much as an increased flow of air lowers the pressure-difference developed by a fan.

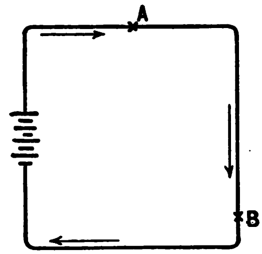


Fig. 40.

Power expended in a portion of a circuit.—Let P' be that part of the total power delivered by a generator, which is expended in a given portion between A and B, Fig. 40, of a circuit, and let i be the current flowing. Then

$$P' = E'i \tag{33b}$$

in which E' is the electromotive force between the points A and B.

48. Ohm's law.—An important relation between electromotive force, current strength, and resistance exists *in case of a circuit in which all the power delivered by the generator is used in the generation of heat in accordance with Joule's Law.* Thus the power delivered by a generator is Ei, and the rate of generation of heat in the circuit is Ri^2. When the whole of Ei is used in the generation of heat we have $Ei = Ri^2$ or

$$E = Ri \tag{34a}$$

or

$$i = \frac{E}{R} \tag{35a}$$

This relation was established by Ohm in 1827 and is called Ohm's Law.

Ohm's Law applied to a portion of a circuit.—The power expended in the portion between AB of the circuit shown in Fig. 40, according to equation (33*b*), is $E'i$. If all this power is used in the generation of heat in this portion of the circuit then $E'i = R'i^2$ or

$$E' = R'i \qquad (34b)$$

or

$$i = \frac{E'}{R'} \qquad (35b)$$

49. Units of electromotive force. The volt.—The c.g.s. unit of electromotive force is the electromotive force which does work at the rate of one erg per second in producing one c.g.s. unit of current in a circuit; or, it is the electromotive force which produces one c.g.s. unit of current in a c.g.s. unit of resistance.

The *volt* is the electromotive force which does work at the rate of one *watt* (one joule per second) in producing a current of one ampere in a circuit; or, it is the electromotive force which produces one ampere of current in a resistance of one ohm. The volt is equal to 10^8 c.g.s. units of electromotive force.

50. Measurement of electromotive force.—Ohm's Law affords a convenient and easy method for the measurement of electromotive force. The method consists in measuring the current which the given electromotive force can produce in a circuit of known resistance and computing the electromotive force by means of equation (34). If the given electromotive force cannot produce the current without being reduced in value then a modification of this method is used as explained in Chapter VI.

BRANCHED CIRCUITS.

SERIES AND PARALLEL CONNECTIONS.

51. Series and parallel connections.—When two portions of an electric circuit are so connected that the entire current in the circuit passes through both portions, the portions are said to be connected in *series.* When two portions of an electric circuit are so connected that the current in the circuit divides and a part of it flows through each portion, the portions are said to be connected in *parallel.*

Examples.—The ordinary arc lamps which are used to light city streets are connected in series and the entire current delivered by the lighting dynamo passes through each lamp. On the other hand, if the electromotive force of the dynamo is 4,000 volts and if there are 80 lamps in series the electromotive force between the terminals of each lamp is 50 volts. *The electromotive force of a generator is subdivided among a number of things connected in series.*

The ordinary glow lamps which are used for house lighting are connected in parallel between copper mains which lead out from the terminals of the dynamo. Except for a slight loss of electromotive force in the mains, the full electromotive force of the dynamo acts upon each lamp. On the other hand if each lamp takes one ampere of current and if there are 100 lamps the total current delivered by the dynamo will be 100 amperes. *The current delivered by a generator is subdivided among a number of things connected in parallel.*

When a number of separate resistances are connected in series the total resistance of the combination is the sum of the individual resistances.

52. Problem.—*To determine the current in each of two branches of a circuit, in terms of the total current and of the resistances of the respective branches.*

The solution of this problem is based upon two principles as follows:

(*a*) The current in the undivided part of a circuit is equal to the sum of the currents in the respective branches into which the circuit divides. (Kirchhoff's Law.)

(*b*) Let A and B, Figs. 41 and 42, be the points at which two or more branches of a circuit unite. The product of the resistance r of any one of the branches into the current i flowing in the branch is equal to the electromotive force between A and B. Therefore the product ri has the same value for every branch terminating in the points A and B.

Consider a circuit which branches at the points A and B, Fig. 41. Let i be the current in the undivided part of the circuit.

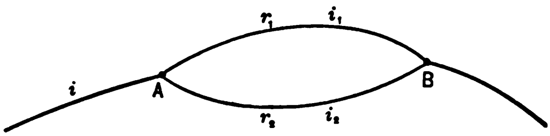


Fig. 41.

Let i_1 be the current in the upper branch and r_1 its resistance; and let i_2 be the current in the lower branch and r_2 its resistance. Then from the above principles we have:

$$i = i_1 + i_2$$

and

$$r_1 i_1 = r_2 i_2$$

Solving these equations for i_1 and i_2 we have

$$i_1 = \frac{r_2 i}{r_1 + r_2} \tag{36}$$

$$i_2 = \frac{r_1 i}{r_1 + r_2} \tag{37}$$

58. Combined resistance of a number of branches of a circuit.— Let A and B, Fig. 42, be the points at which a circuit divides

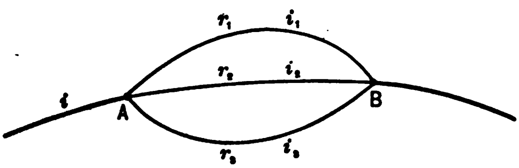


Fig. 42.

number of branches. The combined resistance R of these hes is defined as that resistance which, if connected between

A and B in place of the branches, would carry the whole current i without changing the electromotive force between A and B. That is,

$$i = \frac{E}{R}$$

Furthermore, from Art. 52 (*b*), we have:

$$i_1 = \frac{E}{r_1}$$

$$i_2 = \frac{E}{r_2}$$

$$i_3 = \frac{E}{r_3}$$

in which E is the electromotive force between A and B. Substituting these values of i, i_1, i_2 and i_3 in the equation

$$i = i_1 + i_2 + i_3$$

we have

$$\frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3}$$

or

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \tag{38}$$

54. The use of shunts with galvanometers.—In the use of a galvanometer or other current-measuring instrument it is fre-

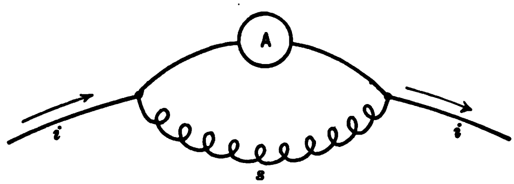


Fig. 43.

quently undesirable to send the whole current through the instrument. In such a case a definite fractional part of the current is diverted by making the instrument one of two branches of the circuit, as shown in Fig. 43, in which A represents the instrument and s the auxiliary branch. This auxiliary branch is called a *shunt*.

Let A represent the resistance of the instrument and s the resistance of the shunt. Then, according to equation (36), the fractional part $\frac{s}{s + A}$ of the total current i passes through A.

CHAPTER IV.

INDUCED ELECTROMOTIVE FORCE.

THE DYNAMO.

55. Faraday's discovery. Lenz's Law.—Faraday discovered in 1831 that a momentary electric current is produced in a coil of wire when a magnet is *pushed into* or *withdrawn from* the opening of the coil, or when an iron rod which passes through the coil is *magnetized* or *demagnetized.* The motion of the magnet in the first case, or the varying magnetism of the iron rod in the second case, produces a momentary electromotive force which in its turn produces a momentary current in the coil. Electromotive force and electric current produced in this way are called *induced electromotive force* and *induced current.*

Lenz's Law.—An induced current always opposes the action which produces it, and the work done in overcoming this opposition goes to maintain the induced current.

Examples.—The current induced in a coil when a magnet is pushed *into* the coil is in such a direction as to tend to push the magnet *out of* the coil.

When an iron rod wound with wire is magnetized the current induced in the winding of wire opposes the magnetization and more work is required to magnetize the rod than would be required if the induced current did not exist. This additional work is that which produces the induced current.

56. Electromotive force induced in a straight wire moving sidewise across a uniform magnetic field.—By the help of Lenz's Law, the fundamental law of induced electromotive force may be derived. For this purpose let us consider the simplest case, namely, the electromotive force induced in a straight wire moving sidewise across a uniform magnetic field.

The straight wire BB', Fig. 44, slides sidewise at velocity v centimeters per second along two straight wires AB and $A'B'$ distant l centimeters from each other. The wires AB and $A'B'$ are connected at AA' so that $ABB'A'$ is a closed circuit. The

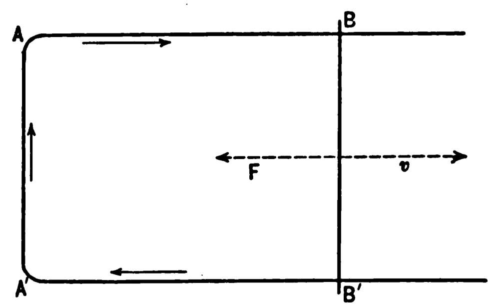


Fig. 44.

whole arrangement is placed in a uniform magnetic field of intensity f, the direction of the field being perpendicular to the plane $ABA'B'$ and towards the reader.

The motion of the wire BB' induces in it an electromotive force E which produces in the circuit $ABB'A'$ a current i; and because of this current the magnetic field pushes the wire BB' sidewise with a force F. This force is, by Lenz's Law, opposed to the velocity v as indicated by the arrow F in Fig. 44.

The rate at which work is done in moving the wire in opposition to the force F is Fv, and the rate at which work is delivered to the circuit in the maintenance of the current i is Ei. Therefore from Lenz's Law we have:

$$Fv = Ei \qquad (a)$$

and from equation (11) Art. 30, we have

$$F = ilf \qquad (b)$$

Substituting this value of F in equation (a) we have:

$$E = lfv \qquad (39)$$

That is, *the electromotive force induced in a wire l centimeters long moving sidewise at a velocity of v centimeters per second across a uniform magnetic field of intensity f is equal to the product lfv.* This product expresses the induced electromotive force in c.g.s. units.

Expression of induced electromotive force in terms of lines of force cut per second.—During a time interval Δt the sliding piece BB', Fig. 44, moves over a distance $v \cdot \Delta t$ and sweeps across $lv \cdot \Delta t$ square centimeters of area. Multiplying this area by f gives the number of lines of force $\Delta\Phi$ which pass through the area, according to equation (9), and this is the number of lines of force cut by BB' during the time Δt. Therefore $\Delta\Phi = lfv\Delta t$ or

$$\frac{\Delta\Phi}{\Delta t} = lfv = E$$

That is, the *induced electromotive force is equal to the rate at which the moving wire cuts lines of force.* This result may be easily established for any wire, straight or curved, moving in any manner in any magnetic field uniform or nonuniform.

Expression for induced electromotive force in terms of rate of change of flux through a circuit.—Let Φ be the total magnetic flux through the circuit $ABB'A'$, Fig. 44. Then $\Delta\Phi$, in the above discussion, is the increment of Φ during the time interval Δt, and the rate at which BB' cuts lines of force is the rate of increase of Φ. Therefore *the induced electromotive force in a circuit is equal to the rate of change of the magnetic flux through the circuit,* or

$$E = -\frac{d\Phi}{dt} \tag{40}$$

Experiment shows this equation to be true in every case, be the change of magnetic flux due to motion or to varying strength of the magnetic field.

The negative sign in equation (40) has no immediate importance. It is chosen in accordance with the following convention. A *right-handed screw* with its axis parallel to the magnetic field f (directed towards the reader in Fig. 44) would have to be turned in a direction *opposite* to the flow of induced current produced by increasing flux in order that the screw might move in the direction of f. It is therefore convenient to look upon the induced current or the induced electromotive force as negative when $\frac{d\Phi}{dt}$ is positive.

When the magnetic flux through the opening of a coil of wire changes, the electromotive force induced in each turn of wire is $-\frac{d\Phi}{dt}$ and the total electromotive force induced in the coil is

$$E = -Z\frac{d\Phi}{dt} \tag{40a}$$

in which Z is the number of turns of wire in the coil.

57. The dynamo is a machine by means of which mechanical power may be used for the production and maintenance of electric current. Precisely the same machine, electric current being supplied, may be used for the generation of mechanical power. When used for this latter purpose the dynamo is called an *electric motor*.

When used as a generator of electric current the action of the dynamo is essentially as follows: An electric wire is moved sidewise across a magnetic field in the direction *opposite* to the side-push upon it. In this case the work done in moving the wire goes to maintain the current.

When used as a motor the action of the dynamo is essentially as follows: A wire moving sidewise across a magnetic field has current forced through it in the direction *opposite* to the induced electromotive force. In this case the work done in maintaining the current, by the outside agent which generates the current, goes to maintain the motion.

58. The alternating current dynamo.—This dynamo consists essentially of a coil of wire which is moved near a magnet in such a way that the magnetic flux from the magnet passes through the coil first in one direction and then in the other direction repeatedly. This induces an electromotive force in the coil in one direction while the magnetic flux is increasing, and in the other direction while the flux is decreasing. This *alternating electromotive force* produces an *alternating current* in the coil and in any outside receiving circuit which is connected to the terminals of the coil. The alternating current dynamo is usually called, simply, an *alternator*.

A common type of alternator consists of a multipolar electromagnet (*the field magnet*), of which the poles project radially in-

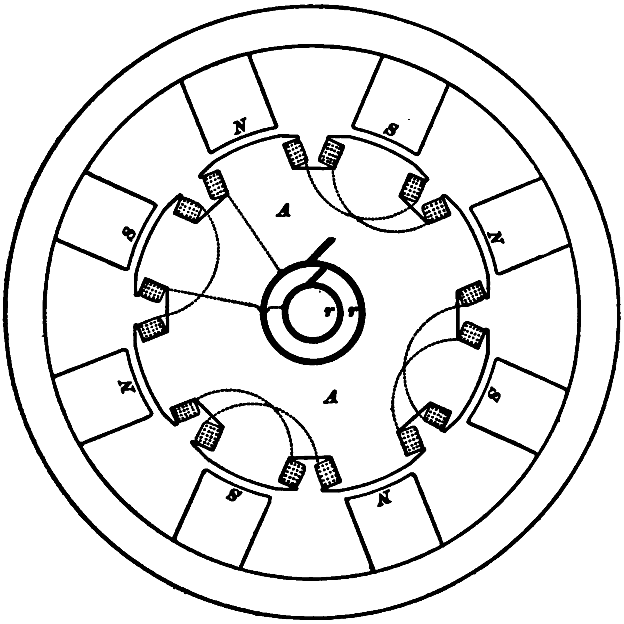


Fig. 45.

wards towards the passing teeth of a rotating toothed iron wheel, *A* (*the armature*), as shown in Fig. 45. On the armature shaft, at one end of the armature, are fixed two insulated metal rings, *rr* (*collecting rings*), upon which metal springs (*brushes*) rub. The ends of the armature wire are soldered to the respective collecting rings, and the wire is wound around the armature teeth, in opposite directions around adjacent teeth. The dotted lines show the connections between the coils and to the collecting rings. The terminals of the external circuit which is to receive current from the machine are connected to the brushes. The field magnet of the alternator is magnetized (*excited*) by a continuous or steady current from some independent source. The armature core *A*, Fig. 45, is built up of stampings of thin sheet iron to prevent eddy currents as explained in Art. 62.

Alternators used in practice give from 50 to 250 or more reversals of current per second.

59. The direct current dynamo is a device for the production of a continuous or steady current by the motion of wires in a magnetic field. This machine is somewhat more complicated than the alternator.

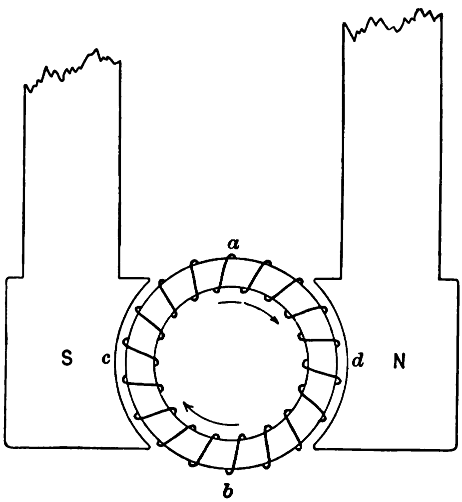


Fig. 46.

An iron ring *ab*, Fig. 46, which is built up of sheet-iron stampings, is wound uniformly with insulated wire the ends of which are spliced and soldered together so that the winding is endless. This iron ring with its winding is called an *armature*. It rotates between the poles of a strong *field magnet* as indicated by the curved arrows.

The wires on the outside of the armature have electromotive forces induced in them as they sweep across the pole faces of the field magnet. These electromotive forces cannot produce current in the endless wire that is wound on the armature for the reason that exactly equal and opposite electromotive forces are induced on the opposite sides *c* and *d* of the ring as shown

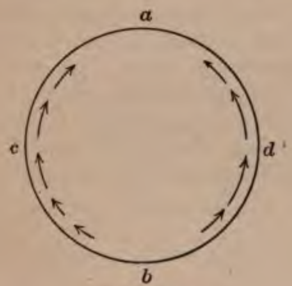


Fig. 47.

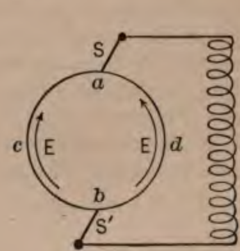


Fig. 48.

schematically in Fig. 47, in which figure the circle represents the endless wire which is wound on the ring. A steady, or very nearly steady current can, however, be taken from the winding of the ring through an outside receiving circuit *l*, Fig. 48, by keeping the terminals of this circuit in metallic contact with the windings on the ring at *a* and *b*. For this purpose the insulation may be removed from the outer portions of the windings on the ring, or from every second turn, or every third turn, etc., and metallic springs *SS*, Fig. 48, may be arranged to rub at *a* and *b* as the ring rotates. In practice, wires are soldered to the various turns, or to every second turn, or every third turn, etc., of the ring winding, and led down to insulated copper bars near the axis of rotation. Sliding contact is then maintained with these bars, instead of with the turns of wire at *a* and *b* directly. This set of

Fig. 49.

copper bars constitutes what is called the *commutator*, an end view of which is shown in Fig. 49.

The field magnet of the direct current dynamo is usually excited by current taken from the machine itself. The type of armature here described is called the *ring armature*. Another type of armature, called the *drum armature*, is frequently used.

The fundamental equation of the direct current dynamo.—Let Φ be the number of lines of flux which emanate from the north pole of the field magnet, pass into the armature ring, and then pass out into the south pole of the field magnet. Let Z be the number of turns of wire on the ring, let n revolutions per second be the speed of the armature, and let E be the electromotive force between the points a and b. Consider a given turn of wire on the ring. While this turn is moving from a to b, Fig. 46, the outer part of the turn cuts all of the Φ lines of force which emanate from the north pole of the field magnet. The time required for the turn of wire to move from a to b is the time of half a revolution or $\frac{1}{2n}$ seconds; therefore the turn cuts lines of force at the average rate $\Phi \div \frac{1}{2n}$, or $2n\Phi$, lines per second, which, by equation (40) is equal to the average electromotive force induced in the given turn while it is moving from a to b. There are $\frac{Z}{2}$ turns of wire in series between a and b and the average electromotive force in each is $2n\Phi$ so that the total electromotive force between a and b is $\frac{Z}{2} \times 2n\Phi$. That is

$$E = \Phi Z n$$

or

$$E_{\text{volts}} = \frac{\Phi Z n}{10^8} \qquad (41)$$

60. The induction coil.—An iron rod wound with insulated wire may be repeatedly magnetized and demagnetized by repeatedly connecting and disconnecting a battery to the winding. The increasing and decreasing magnetic flux thus produced through the rod may be utilized to induce electromotive force in an auxiliary coil of wire wound on the rod. Such an arrangement is called an *induction coil*. The winding through which the magnetizing current from the battery flows is called the *primary coil* and the auxiliary winding is called the *secondary coil*. The iron rod is usually made of a bundle of fine iron wires to prevent eddy currents, as explained in Art. 62.

When the iron core is magnetized a pulse of electromotive force is induced in the secondary coil, and when the core is demagnetized a reversed pulse of electromotive force is induced in the secondary coil. These impulsive electromotive forces may be made to reach very great values, hundreds of thousands of volts, by making the secondary coil of *many turns of wire*, and by providing for *the quickest possible magnetization or demagnetization of the core.*

A battery, or any ordinary current generator, does not magnetize a core very quickly when connected to a winding of wire, in fact, a very considerable fraction of a second is usually required for the core to become magnetized. Therefore, during the magnet-

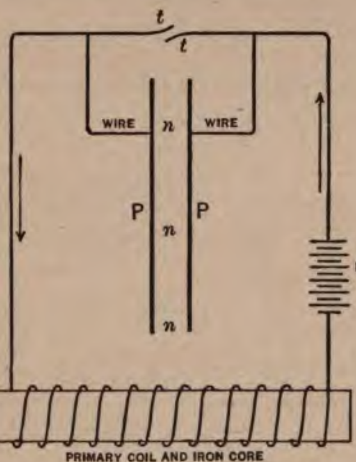


Fig. 50.

ization of the iron core of an induction coil, the electromotive force induced in the secondary coil is *a comparatively weak pulse of long duration.*

On the other hand, proper arrangements permit of an extremely quick demagnetization of the iron core of an induction coil when the battery is disconnected from the primary winding, and this quick demagnetization induces in the secondary coil *an intense pulse of electromotive force of short duration.*

The quick demagnetization of the iron core of an induction coil is accomplished as follows: Fig. 50 shows the connections of a battery to the primary coil. The secondary coil is omitted for the sake of clearness. The battery is connected and disconnected by *making* and *breaking* contact between the metal terminals *tt*. *PP* are large metal plates (sheets of tin foil), separated by an insulator *n*, such as waxed paper, and connected to the terminals *tt* as shown. When the points *tt* are connected the core is slowly magnetized by the current from the battery. When *tt* are disconnected the current persists in flowing for a short interval of time. This persisting current flows into the plates *PP* and the electric charge thus accumulated on the plates *PP* surges back through the circuit as a reversed current and demagnetizes the iron core.

61. The alternating current transformer consists of two coils of wire, a *primary* coil and a *secondary* coil, wound upon an iron

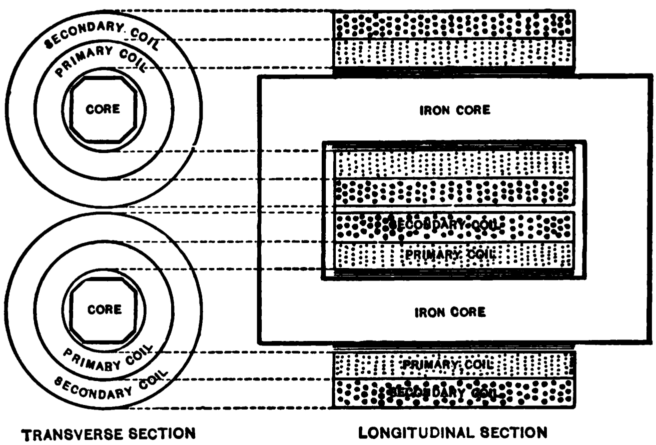


Fig. 51.

core. This iron core usually forms a complete magnetic circuit, as shown in Fig. 51, which represents a commercial type of transformer.

The induction coil and the alternating current transformer are identical except that the iron core of the induction coil is not a closed magnetic circuit, but has magnet poles at its ends. The effect of these magnet poles is to facilitate the quick demagnetization of the core when the primary circuit of the induction coil is broken.

The action of the transformer.—Alternating current is supplied to the primary of the transformer. This alternating current produces rapid reversals of the magnetization of the iron core. These magnetic reversals induce alternating electromotive force in the secondary coil, and the secondary coil supplies alternating current to a receiving circuit.

Step-up and step-down transformation.—Usually, one coil of a transformer has many more turns of wire than the other. The coil of many turns may take a small current at high electromotive force when the coil of few turns will deliver a large current at low electromotive force. This is called *step-down* transformation.

The coil of few turns may take a large current at low electromotive force when the coil of many turns will deliver a small current at high electromotive force. This is called *step-up* transformation.

The object of step-up and of step-down transformation is the following : The transmission of a given amount of power electrically may be accomplished by a large current at low electromotive force or by a small current at high electromotive force. In the former case very large and expensive transmission wires must be used or the loss of power in the transmission wires will be excessive. In the latter case comparatively small and inexpensive transmission wires may be used without involving excessive loss of power. Therefore high electromotive force is a practical necessity in the long distance transmission of power. The user of electric power must, however, be supplied with current at low electromotive force partly on account of the danger involved in the use of high electromotive force and partly on account of the fact that many types of electrical apparatus cannot be operated satisfactorily with high electromotive force ; likewise it is inconvenient and dangerous to generate excessively high electromotive force in a complicated machine like an alternator which must be cared for by an attendant. These difficulties are met by using a transformer for step-up transformation at the generating station and another transformer for step-down transformation at the receiving station.

The theory of the action of the transformer.—In the following discussion Z' represents the number of turns of wire in the primary coil, and Z'' the number of turns of wire in the secondary coil. The effect of the resistance of the coils, which is usually quite small, is ignored.

Ratio of primary current to secondary current.—Aside from resistance, the only thing which opposes the flow of current through the primary coil is the *reacting* electromotive force in the primary coil induced by the reversals of magnetization of the core. The greater the range of this magnetization the greater the value of this reacting electromotive force. *The combined magnetizing action of primary and secondary coils is always such as to magnetize the core to that degree which will make the reacting electromotive force in the primary coil equal to the electromotive force of the dynamo which is forcing current through the primary coil.* Action is equal to reaction.

When the secondary coil is on open circuit, just enough current flows through the primary coil to produce the degree of magnetization above specified. Let this value of the primary current, which is called the *magnetizing current* of the transformer, be represented by m.

When a current I'' is taken from the secondary coil, an *additional* current I', over and above the current m, flows through the primary coil. The current m still suffices to magnetize the core, and *the magnetizing action of I'' is exactly neutralized by the equal and opposite magnetizing action of I'.* The magnetizing action of I'' is measured by the product $Z''I''$, and the magnetizing action of I' is measured by the product $Z'I'$, so that, ignoring algebraic signs, we have

$$Z'I' = Z''I''$$

or

$$\frac{I'}{I''} = \frac{Z''}{Z'} \tag{42}$$

Ratio of primary electromotive force to secondary electromotive force.—The fluctuating magnetization of the core of a transformer induces a certain electromotive force e in each turn of wire surrounding the core. Therefore the total electromotive force induced in the primary coil is $Z'e$. This is the reacting electromotive force in the primary coil and it is equal, as explained above, to the outside electromotive force E' which is pushing current through the primary coil, so that:

$$E' = Z'e \tag{a}$$

Similarly, the total electromotive force E'', induced in the secondary coil is

$$E'' = Z''e \tag{b}$$

Therefore

$$\frac{E'}{E''} = \frac{Z'}{Z''} \tag{43}$$

62. Eddy currents. Lamination.—When a piece of iron is magnetized or demagnetized, the changing magnetic flux through the central portions of the iron induces electromotive forces in the surrounding portions, and these electromotive forces produce what are called *eddy currents*. Eddy currents are also produced in a mass of metal which is near a moving magnet, or which moves in the neighborhood of a stationary magnet.

Lamination.—Those parts of electrical machinery which are subject to rapid and frequent changes of magnetization are built up of thin sheets of iron, or of iron wire, so as to leave the iron *continuous* in the direction of the magnetization but *discontinuous* in the direction in which the eddy currents tend to flow. Such a mass of iron is said to be *laminated.* The iron parts of dynamo armatures and of transformers are laminated.

Examples of eddy currents.—A bundle of iron wires surrounded by a winding of wire is magnetized, say, in one second when the winding is connected to a battery, and demagnetized in a much shorter time when the battery is disconnected. A solid iron rod of the same size would require perhaps nine or ten seconds to be magnetized by the same coil and battery, and upon disconnecting the battery the solid rod would require nine or nine and a half seconds to lose its magnetism. *The eddy currents in the solid rod oppose magnetization while the rod is being magnetized, and tend to keep up the magnetization while the rod is being demagnetized.* (Lenz's Law.)

A suspended magnet set swinging is quickly brought to rest if it is surrounded by a massive ring of copper, because the eddy currents induced in the copper by the moving magnet act upon the magnet with a force which is at each instant opposed to the motion. (Lenz's Law.)

A sheet of copper suddenly thrust between the poles of a strong magnet behaves as if it were moving in a viscid liquid. Eddy currents are induced in the copper; because of these currents the magnet exerts a force upon the copper, and this force is always opposed to the motion. (Lenz's Law.)

CHAPTER V.

ELECTROLYSIS. BATTERIES.

63. Electrolysis.—When the electric current passes through a conducting liquid which is not a chemical element, the liquid is decomposed. For example, molten NaCl is broken up into metallic sodium and chlorine by the electric current. Many liquids (for example, pure water, pure alcohol, etc.) scarcely conduct the electric current at all. When salts or acids are dissolved in such liquids they become conductors. In such cases it is the dissolved substance which is decomposed by the current. The products of decomposition of the dissolved substance sometimes react upon the solvent, however, thus decomposing it. For example, in an aqueous solution, H_2SO_4 is broken up into H and SO_4 by the current. The H appears at one electrode and escapes as a gas, and the SO_4 appears at the other electrode, where it acts upon the water, forming H_2SO_4, which goes into solution, and O, which escapes as a gas.

The decomposition of a liquid by the electric current is called *electrolysis*, and liquids which are decomposed by the current are called *electrolytes*. Solutions of salts and acids generally are electrolytes. Electrolytes generally are of high resistance as compared with metals, so that electrolysis is usually carried out in a vessel provided with two large conducting plates of metal or carbon called *electrodes* (Fig. 52). Such an arrangement is called an electrolytic cell. The electrode at which the current enters

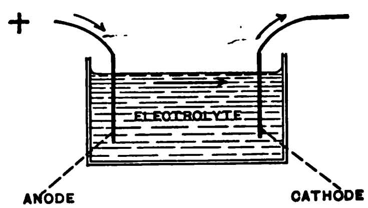


Fig. 52.

the cell is called the *anode*. The electrode at which the current leaves the cell is called the *cathode*.

Consider an electrolyte, a solution of $CuSO_4$, for example. During electrolysis, this salt is in part decomposed, copper being deposited upon the cathode, and SO_4 being liberated at the anode, so that, on the whole, the solution becomes less concentrated. Hittorf, taking precautions against the mixing of the solution by liquid currents and diffusion, found that this weakening of the solution of an electrolyte occurs wholly in the immediate neighborhood of the electrodes.

This weakening of an electrolyte is easily shown by the upward streaming of the weakened solution along the faces of the electrodes. In case the solution becomes denser because of secondary reactions, as near a copper anode in a solution of $CuSO_4$, the solution can be seen to stream downwards.

64. The laws of electrolysis.—*First law. The amount of metal deposited electrolytically by a current is proportional to the strength of the current and to the time.* That is:

$$M = kIt \tag{44}$$

in which M is the mass of metal, in grams, deposited in t seconds by a current I, and k is a constant for a given metal. This quantity k is called the *electro-chemical equivalent* of the metal.*

Electro-chemical equivalents are ordinarily specified in grams of metal deposited per ampere of current per second.

Second law.—The electro-chemical equivalents of the various metals (and of other elements which can form an ion of an electrolyte) *are proportional to the quotients of their atomic weights divided by their valencies.*

A metal which has two valencies has two values for its electro-

* The product It, as will be shown in Chapter X., is the electric charge which has passed through the cell so that this first law may be stated thus: *The deposit of metal is proportional to the charge which has passed through the electrolytic cell.* Electro-chemical equivalents, in terms of the charge transferred, are expressed in *grams per coulomb*.

chemical equivalent. Thus 1½ times as much iron is deposited from a solution of a *ferrous* salt, as is deposited from a solution of a *ferric* salt, provided of course that the deposition is not complicated by secondary reactions at the cathode.

TABLE.

Electrochemical equivalents.

Element.	Valency.	Equivalent in grams per coulomb.	Element.	Valency.	Equivalent in grams per coulomb.
Aluminium,	III.	0.00009450	Potassium,	I.	0.0004054
Copper,	I.	0.00065420	Silver,	I.	0.001118
Copper,	II.	0.00032710	Sodium,	I.	0.0002387
Gold,	III.	0.00067910	Tin,	II.	0.0006116
Hydrogen,	I.	0.00001038	Tin,	IV.	0.0003058
Iron,	II.	0.0002909	Zinc,	II.	0.0003370
Iron,	III.	0.0001935	Bromine,	I.	0.0008282
Lead,	II.	0.001072	Chlorine,	I.	0.0003672
Magnesium,	II.	0.0001243	Iodine,	I.	0.001314
Mercury,	I.	0.002075	Nitrogen,	III.	0.00004850
Mercury,	II.	0.001037	Oxygen,	II.	0.00008287
Nickel,	II.	0.0003043			

65. The dissociation theory of electrolysis.—An electrolytic salt or acid when in solution, or when melted, is thought to be more or less dissociated into what are called its *ions*. For example, the ions of $CuSO_4$ are Cu (atoms) and SO_4; the ions of NaCl are Na (atoms) and Cl (atoms). These ions are supposed to be electrically charged (see Chapter X.) and to wander about through the solution. When an electric current passes through the electrolyte, the positively charged ions (cations) move towards the cathode, where they part with their positive charges and are deposited as hydrogen or metal, as the case may be; and the negatively charged ions (anions) move towards the anode, where they part with their negative charges. This movement of positively and negatively charged ions constitutes the electric current in the electrolyte.

The ions of a given substance have equal charges, so that to transfer a given charge, *It*, through the electrolytic cell, a proportional number of ions must move through the solution and be deposited upon, say, the cathode (*First law*). The charge of an ion is proportional to its valency. Thus the copper ion in a solution of $CuCl_2$ has twice as much charge as the hydrogen ion, for example, in a solution of HCl. Half as many copper ions as hydrogen ions are deposited, therefore, by a given current in a given time. The atom of copper weighs 63.3 times as much as the atom of hydrogen, so that the electro-chemical equivalent of copper is to the electro-chemical of hydrogen as $\frac{63.3}{2} : 1$ (*Second law*).

66. Hittorf's ratio.—During the electrolysis of a solution of a salt, $CuSO_4$ for example, the amount of the salt in the solution is diminished if there are no secondary reactions at the electrodes. After the electrolysis has been kept up for some time a certain total diminution in the amount of dissolved salt will be produced. Let a be the diminution of dissolved salt in the neighborhood of the anode and c the diminution of dissolved salt in the neighborhood of the cathode. *The ratio a/c, called Hittorf's ratio, has a definite characteristic value for every electrolytic salt or acid in dilute solution.*

In most cases, for example in the electrolysis of $CuSO_4$, the anion reacts upon the solvent and goes into solution. The solution near the anode is then no longer a simple solution of the original salt, but contains, in addition, the products resulting from the breaking up of the anion, or the products resulting from the action of the anion upon the solvent or upon the material of the anode. Thus, if a copper anode is used in the electrolysis of $CuSO_4$, the anion SO_4 attacks the copper anode, forming $CuSO_4$, which goes into solution. In this case, the solution in the neighborhood of the anode would have an excess of $CuSO_4$ exactly equal to the diminution of $CuSO_4$ in the neighborhood of the cathode.

67. Proposition.—Hittorf's ratio for a given salt or acid is equal to the ratio of the velocities of the cations and anions, respectively, as they move through the electrolyte carrying the current.

Consider the electrolysis of a solution of $CuSO_4$. Suppose the electrolysis to have continued until 159.3 grams of $CuSO_4$ (one gram-molecule)* have been decomposed, 63.3 grams of copper being deposited on the cathode, and 96 grams of SO_4 being liberated at the anode.

If we imagine the current through the electrolyte to depend entirely upon the movement of copper ions, the SO_4 ions being supposed stationary throughout the middle portions of the electrolyte, then the solution near the anode will have become deficient in $CuSO_4$ by the whole amount of 159.3 grams, and the solution will be unchanged in strength everywhere else.

*The molecular weight of $CuSO_4$ is 159.3 and 159.3 grams of this salt is called a *gram-molecule* of it. The molecular weight of NaCl is 58.5 and 58.5 grams of this salt is called a gram-molecule of it, and so on.

If we imagine the whole current through the electrolyte to depend entirely upon the movement of SO_4 ions, the copper ions being supposed stationary throughout the middle portions of the electrolyte, then the solution near the cathode will have become deficient in $CuSO_4$ by the whole amount of 159.3 grams, and the solution will be unchanged in strength everywhere else.

If the velocity of the copper ions is to the velocity of the SO_4 ions as $n : 1$, then $\frac{n}{n+1}$ of the current may be attributed to the movement of copper ions, and $\frac{1}{n+1}$ of the current may be attributed to the movement of SO_4 ions.

On account of the movement of copper ions the solution in the neighborhood of the anode will have become deficient in $CuSO_4$ by the amount of $\frac{n}{n+1} \times 159.3$ grams, since 159.3 is the deficiency at the anode which would be produced if the whole current were due to the movement of copper ions.

Similarly, $\frac{1}{n+1} \times 159.3$ grams is the deficiency in $CuSO_4$ in the neighborhood of the cathode, on account of the movement of the SO_4 ions. Therefore the ratio of these deficiencies is n.

It is to be kept in mind in this discussion that as the SO_4 ions move through the electrolyte away from the vicinity of the cathode, the same number of Cu ions are deposited at the cathode without having to travel through the middle portions of the electrolyte. The same is true of the liberation of SO_4 ions at the anode as the copper ions move away from the vicinity of the anode towards the cathode. Thus if the copper ions and SO_4 ions were to move through the solution at the same velocity, half of the copper which is deposited on the cathode would travel through the solution from the vicinity of the anode, and half would come from the immediate vicinity of the cathode, because of the movement of SO_4 ions away from that region towards the anode.

68. Relative velocity of ions.—Let the velocity of hydrogen ions under given conditions be taken as unity. Then the velocity of SO_4 ions under the same conditions is found by multiplying this unit velocity by Hittorf's ratio for H_2SO_4. The velocity of copper ions may then be found by dividing the velocity of SO_4 ions by Hittorf's ratio for $CuSO_4$, the velocity of chlorine ions by multiplying the unit velocity of hydrogen ions by Hittorf's ratio for HCl, etc. In this way values may be calculated for the relative ionic velocities of various substances.

69. Molecular conductivity. Ratio of dissociation.—Let c be the concentration of an electrolyte in gram-molecules of salt per liter of solution. The number of actual molecules of salt per cubic centimeter of solution is proportional to c. Let $1 : \mu$ be the ratio of the whole number of molecules of salt per cubic centimeter of solution, to the number of molecules which have been dissociated into ions. The number of ions per cubic centimeter of solution is then proportional to μc.

The amount of current carried by a given electrolyte is proportional to the number of ions per c.c., everything else remaining the same. But anything which increases current with constant electromotive force must decrease resistance or increase conductivity in the same proportion. Therefore the specific conductivity, k, of a given electrolyte is proportional to μc. In very dilute solution μ approaches unity; *i. e.*, all the molecules of the salt are dissociated, so that μc then becomes c, k is then proportional to c, and the ratio k/c is a constant. This constant is called the *molecular conductivity* of the electrolyte. Representing the molecular conductivity by m, we have, for very dilute solutions, $k/c = m$, or for solutions of ordinary concentration $k/\mu c = m$, or:

$$k = \mu c m$$

The values of k and c are easily determined in every case, the value of m is the ratio k/c, when the solution is very dilute, and is thus easily determined. Therefore the ratio of dissociation, μ, of a given electrolyte may be calculated when m has been determined in this way for a dilute solution of the electrolyte and the values of k and c have been determined for the given solution. There is an independent method for determining the values of μ; namely, by observing the freezing point of the electrolyte. This method gives in every case the same values of μ as the electrical method.

70. Work spent in forcing an electric current through an electrolytic cell.—This work consists mainly * of three parts.

(*a*) The work which appears as heat throughout the electrolyte. The rate at which work is so spent, or the rate of generation of heat throughout the electrolyte, is accurately *proportional to the square of the current.* The proportionality factor, R, is called the resistance of the electrolyte.

(*b*) The work which appears as heat *at the electrodes* as the ions are deposited. The rate at which work is so spent increases very rapidly for a few moments after the current is started, and then remains nearly constant, for a given value of the current, etc. This rate of expenditure of work is thus practically a function, $\phi(i)$, of the current after the current is once well established.

(*c*) The chemical decomposition of the electrolyte by the current *requires* work, and the reaction of the liberated ions upon the solvent or upon the electrodes is often a *source* of work. The net rate at which work is so spent is accurately *proportional to the current;* it may be either *positive* or *negative* according as the work required to decompose the dissolved salt or acid is *greater* or *less* than the work generated by the action of the liberated ions upon the solvent and upon the electrodes.

* We here ignore thermo-electromotive forces, and electromotive forces at places where the electrolyte changes in concentration or composition. To consider these secondary effects would carry the subject beyond the limits of this treatise.

71. Energy equation of an electrolytic cell.—Let a current i be forced through an electrolytic cell by an outside agent (battery or dynamo) of which the electromotive force is E. The rate at which this agent does work upon the cell is Ei. The rate at which energy appears as heat throughout the electrolyte is Ri^2. The rate at which energy appears as heat at the electrodes, provided the current has been flowing for some time, is $\phi(i)$. Finally, the rate at which energy is used in bringing about the chemical action is ei. We have, therefore,

$$\left.\begin{aligned} Ei &= Ri^2 + \phi(i) + ei \\ \text{or} \qquad E &= Ri + \frac{\phi(i)}{i} + e \end{aligned}\right\} \tag{45}$$

in which R is the resistance of the electrolytic cell; that is, the proportionality constant mentioned in item (a) Art. 70, e is the proportionality constant mentioned in item (c) Art. 70, and $\phi(i)$ is the function implied in item (b) Art. 70.

The second form of equation (45) shows that the electromotive force, E, of the agent, may be thought of as broken up into three parts, viz.: the part Ri, which is used to overcome the resistance of the electrolyte; the part e, which balances what is called the *counter electromotive force* of the electrolytic cell; and the part $\frac{\phi(i)}{i}$, which overcomes what is sometimes called the *polarization electromotive force* of the cell.

The polarization electromotive force, $\frac{\phi(i)}{i}$, of an electrolytic cell always opposes the current and it manifests itself at both electrodes. At each electrode the polarization electromotive force depends upon the *current per unit area of the electrode*, or, in other words, upon what is called the current density.

Example.—The passage of current through an electrolytic cell having platinum electrodes and containing H_2SO_4 results in the generation of oxygen and hydrogen, so that the chemical action produced is in effect the decomposition of H_2O. The decomposition of one gram of H_2O requires 162×10^9 ergs of energy.

A current of strength i, c.g.s. units, decomposes 0.000933 i grams of water per second, requiring the expenditure of $150 \times 10^6 \times i$ *ergs per second.* This is equal to ei by Art. 70. Therefore e is in this case 150×10^6 c.g.s. units of electromotive force or 1.5 volts.

When the current is small the electromotive force, Ri, required to overcome the resistance of the electrolyte is negligible, and the electromotive force, E, producing the current in the cell becomes $E = \frac{\phi(i)}{i} + e$ from equation (45). The electromotive force required to produce a small current through the electrolytic cell under consideration (solution of H_2SO_4 with platinum electrodes) is observed to be about 2 volts, so that the minimum value of $\frac{\phi(i)}{i}$ seems to be about 0.5 volt.

If a small quantity of free oxygen is in the solution, the hydrogen appearing at the cathode will be reduced to H_2O, and the oxygen in solution will be replenished by the oxygen appearing at the anode, which slowly diffusing through the solution, will keep up the supply of free oxygen at the cathode. Thus a *very small* current may flow through the electrolytic cell without actual decomposition of water and therefore without encountering the opposing electromotive force e.

72. Voltaic cells.—An electrolytic cell, in which the chemical actions brought about by the current are a *source* of energy, is called a *voltaic cell.* For example, an electrolytic cell containing dilute H_2SO_4, and having a zinc anode, helps to maintain a current which passes through it, because of the fact that in the decomposition of H_2SO_4 less energy is required than is furnished by the combination of the SO_4 ions with the zinc of the anode. When the electrodes of such an electrolytic cell are connected by a wire, a current starts and is maintained without the aid of any external agent. A group of cells so connected as to supply current by their common action to a given circuit is termed a *battery.*

73. Energy equation of the voltaic cell.—In case of the voltaic cell (electrolytic cell which maintains the current flowing through it) the electromotive force E equation (45) is zero, and ei being the rate at which energy is *furnished* by the cell changes sign, so that equation (45) becomes

$$\left.\begin{aligned} ei &= Ri^2 + \phi(i) \\ \text{or} \qquad e &= Ri + \frac{\phi(i)}{i} \\ \text{or} \qquad i &= \frac{e - \frac{\phi(i)}{i}}{R} \end{aligned}\right\} \qquad (46)$$

in which ei is the rate at which energy is *furnished* by the chemical actions in the cell, Ri^2 is the rate at which energy appears as heat throughout the circuit, including electrolyte and wire connecting the electrodes, and $\phi(i)$ is the rate at which energy appears as heat at the electrodes as the ions are deposited.

The quantity $e - \frac{\phi(i)}{i}$ is the electromotive force of the cell. The quantity e is the electromotive force that the cell would have if all the energy of the chemical reactions were available in producing current. The quantity e is numerically equal to the ergs of energy developed by the chemical action which takes place during one second when one unit of current is flowing through the cell. This energy may be measured as heat if the given chemical action is made to take place in a calorimeter and thus e may be determined.

For some cells the polarization, $\frac{\phi(i)}{i}$, is quite small when the current is very small. The electromotive force $\left(e - \frac{\phi(i)}{i}\right)$ of such cells when giving a small current, being sensibly equal to e, may be calculated from thermo-chemical data, as explained above.

74. Local action.—In some forms of cell, chemical action goes on independently of the flow of current, when no current is flowing as well as when current is flowing. Such chemical action contributes in no way to the production of current by the cell, and all the energy developed by this chemical action is lost. This chemical action is called *local action*. For example, in a Grenet cell, even when it is being used to furnish a large current, as much as eighty per cent. of the zinc and acid used is lost in local action.

Amalgamation of the zinc electrodes.—Local action seems to be of two distinct kinds: (*a*) The dissolving of the zinc by the electrolyte independently of any electric current whatever, and (*b*) the dissolving of the zinc because of small particles of carbon, or other foreign substance, which cling to the zinc and act as small *cathodes*. Electric currents flow through the electrolyte from the zinc to each small cathode particle and thence to the zinc again through the point of contact of the particle with the zinc. These local electric currents decompose the electrolyte, liberate the anion at the surface of the zinc, and the zinc is thus consumed.

When commercial zinc is dissolved, in sulphuric acid for example, a portion of its impurities are left on its surface and act in the manner above described. Amalgamating the zinc surface with mercury prevents this action almost completely.

75. Forms of voltaic cells. *General statement.*—In most commercial types of cell the anode is made of zinc; and the performance of the cell is entirely independent of the material of the cathode, provided that it is a good electrical conductor, and that it is not attacked chemically by the electrolyte.

In the operation of a cell the electrolyte is decomposed by the current. The anion (SO_4 or chlorine, for example) appears at the zinc anode and combines with the zinc, and the cation (usually hydrogen) appears at the inert cathode. *The available energy of the cell, and therefore its electromotive force, is greatly increased by providing a supply of available oxygen at the cathode which may combine with the hydrogen.*

Example.—A solution of H_2SO_4 with zinc anode and copper cathode is a simple form of cell. The current flowing through the cell liberates SO_4 ions at the zinc anode, and H ions at the copper cathode; and the hydrogen collects in bubbles and passes off. When a solution of potassium bichromate is poured into the cell, hydrogen ceases to be liberated at the copper cathode, being oxidized by oxygen furnished by the bichromate, and an ammeter connected to the cell shows a greatly increased current due to the increased electromotive force of the cell.

The Bunsen cell consists of a zinc anode, a carbon cathode, and an electrolyte of dilute sulphuric acid. The carbon cathode is surrounded by nitric acid which furnishes oxygen for the oxidation of the hydrogen. Mixing the nitric acid with the dilute sulphuric acid would cause rapid wasting of the zinc by local action, and therefore the nitric acid is placed in a porous earthenware cup which stands in the sulphuric acid and contains the carbon cathode. A porous cup does not break the continuity of an electrolyte.

The Grenet cell consists of a zinc anode, a carbon cathode, and an electrolyte consisting of a solution of sulphuric and chromic acids. The chromic acid furnishes oxygen for the oxidation of the hydrogen. The presence of the chromic acid causes the zinc to waste away rapidly by local action even when the zinc is amalgamated, and the chromic acid is frequently contained in a porous cup in which the carbon cathode, also, is placed.

The Leclanché cell consists of a zinc anode, a carbon cathode, and an electrolyte consisting of a solution of ammonium chloride (NH_4Cl). The carbon cathode has packed about it a mixture of powdered manganese dioxide (MnO_2) and powdered coke. When the current flows, chlorine ions appear at the zinc anode, and form $ZnCl_2$. The NH_4 ions appear at the cathode, break up into NH_3 and hydrogen; the NH_3 goes into solution, and the hydrogen combines with oxygen furnished by the MnO_2. The valuable feature of this cell is its freedom from local action.

The Daniell cell consists of a zinc anode in a solution of zinc sulphate, and a copper cathode in a solution of copper sulphate, the two solutions being kept separate by a porous partition. The *gravity cell* is a Daniell cell in which the copper cathode lies near the bottom of the containing vessel. Surrounding this copper cathode is a concentrated solution of copper sulphate on top of which the less dense solution of zinc sulphate floats. The zinc anode is suspended near the top of the containing vessel in the zinc sulphate solution.

When current flows through the Daniell cell, SO_4 ions appear at the zinc anode forming $ZnSO_4$; and copper ions appear at the cathode and are deposited as metallic copper. The chemical action in the Daniell cell is

$$CuSO_4 + Zn = ZnSO_4 + Cu$$

The Clark standard cell.—This cell has a cathode of mercury which lies at the bottom of the glass containing vessel. Over the surface of the mercury is spread a paste made by rubbing up powdered mercurous sulphate with a concentrated solution of zinc sulphate, in which is the zinc anode. When current flows through this cell, SO_4 ions appear at the zinc anode, forming $ZnSO_4$, and zinc ions appear at the mercury cathode and react upon the mercurous sulphate, forming $ZnSO_4$ and metallic mercury. This cell polarizes very much when any considerable current flows through it. With small currents, however, its electromotive force is very nearly constant, varying slightly with temperature, and it serves admirably as a standard of electromotive force.

76. The storage cell.—The chemical action which takes place in a voltaic cell when it is giving current is exactly reversed when current is forced through it in the opposite direction by an external agent. Any voltaic cell, therefore, may in this way be regenerated to a greater or less extent after it has been used for some time in the production of current. In order that the regeneration of the cell may be complete, it is necessary that the cell be free from local action, and that the products resulting from the chemical action which takes place when the cell is producing current be conserved in the electrolyte. A cell in which these conditions are realized is called a *storage cell.*

The lead storage cell.—The electrodes of this cell consist of two massive lead grids, the meshes or interstices of which are filled with a paste of $PbSO_4$ made by mixing litharge, or litharge and red lead, with dilute sulphuric acid. These electrodes are placed in a dilute solution of H_2SO_4. When a current is sent through the cell hydrogen ions appear at one electrode, react upon the $PbSO_4$, forming spongy lead and H_2SO_4 which goes into solution, and SO_4 ions appear at the other electrode and produce the following reaction:

$$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$$

This H_2SO_4 goes into solution, and the peroxide of lead, PbO_2, is left on the grid. When all or most of the $PbSO_4$ on the electrodes has been changed in this way to lead and PbO_2, respectively, the cell is said to be charged, and it may be used as an ordinary voltaic cell, giving a current in the opposite direction to the charging current, until most of the lead and PbO_2 are changed back to $PbSO_4$.

The lead storage cell gives, upon discharging, very nearly as many ampere-hours as are used in charging. But the polarization electromotive force, $\phi(i)/i$, and the electromotive force Ri which is used to overcome the resistance of the cell are always opposed to the current, and the electromotive force of the cell ranges from 2 to 2.3 volts as the cell is charged, and from 2.1 to 1.8 volts as the cell is discharged. The energy efficiency of the cell is ordinarily about 80 per cent.

The active material ($PbSO_4$ paste) in the grid of a storage cell suffers considerable expansion and contraction during charge and discharge. For this reason, mainly, it is necessary to charge the cell always in the same sense. Even then the grid which is the anode during charge, and the cathode during discharge (the positive grid), tends to fall to pieces. The grids must therefore be repaired or replaced from time to time.

*The Edison storage cell** consists of two nickel-plated grids of sheet steel submerged in a solution of caustic potash. The openings in the negative grid (corresponding to the zinc of a primary cell) are packed with a mixture of flake graphite and an iron salt, and the openings in the positive grid are packed with a mixture of flake graphite and a nickel salt. The grids are then subjected to a preliminary treatment so as to reduce the iron salt to finely divided metallic iron and the nickel salt to finely divided nickel peroxide. When the cell gives current oxygen appears at the anode and combines with the finely divided iron, forming iron oxide; and hydrogen appears at the cathode and reduces the nickel peroxide to nickel oxide. When the cell is charged this action is reversed. The flake graphite is inert and serves to increase the electric conductivity of the grids. The electromotive force of this cell ranges from 1 to 1.2 volts.

* See A. E. Kennelly, *Transactions American Institute of Electrical Engineers*, May, 1901.

CHAPTER VI.

ELECTRICAL MEASUREMENTS.

77. Absolute measurements and international standards.—The measurement of an electrical quantity in terms of the mechanical units of length, mass and time is called *absolute* electrical measurement. Absolute electrical measurement requires, in most cases, elaborate apparatus and, unless extreme precautions are taken, is subject to considerable error. Therefore standards of current, of resistance, and of electromotive force have been measured absolutely with extreme care and adopted as *International Standards* and all ordinary electrical measurements consist in the comparison of the quantity to be measured with these standards.

78. Measurement of current by electrolysis. The international standard ampere.—The absolute measurement of current is explained in articles 34 and 37.* When the electrochemical equivalent of a metal has been determined by weighing the metal deposited by an absolutely measured current, then the strength of any current may be easily and accurately measured by weighing the metal deposited by the current during an observed interval of time. The value of the current is calculated from equation (44) which, solved for current, gives:

$$I = \frac{M}{kt}$$

in which M grams is the amount of metal deposited by the current I in t seconds and k is the previously determined electrochemical equivalent of the metal.

* See also *Absolute Measurements*, A. Gray, Vol. II., pp. 364, 398.

The silver voltameter.—The electrolytic deposition of silver from a solution of silver nitrate affords the most accurate measurement of current by the electrolytic method, for the reason that the deposition of this metal is but little affected by secondary actions at the cathode; and, moreover, the electrochemical equivalent of silver is large and the deposit by a given current is correspondingly large.

The usual form of silver voltameter is shown in Fig. 53. The containing vessel is a platinum bowl, which also serves as the cathode. The anode is a plate of pure silver enveloped in filter paper, and the electrolyte is an aqueous solution of pure silver nitrate.

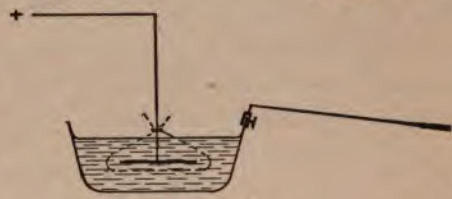


Fig. 53.

The international standard ampere is a current which, when passed through a solution of silver nitrate in accordance with certain specifications,* deposits silver at the rate of 0.001118 gram per second.

The copper voltameter consists of a glass vessel containing an aqueous solution of copper sulphate with electrodes of metallic copper. An aqueous solution of copper sulphate, especially when exposed to the air, slowly dissolves metallic copper, and this slow dissolution of the cathode produces variations in the apparent electrochemical equivalent of copper and leads to slight errors in the determination of current by the copper voltameter.

The water (or sulphuric acid) voltameter consists of a vessel containing dilute sulphuric acid with platinum electrodes. The observed volume of the liberated gases, or the observed loss of weight of the voltameter (gases being thoroughly dried before escape) affords a measure of the current.

* See *Proceedings of the International Electrical Congress*, Chicago, 1893; or *Transactions* of American Institute of Electrical Engineers.

79. The measurement of current by the fall of potential method.—The current, I, to be measured is passed through a known resistance R and the electromotive force (fall of potential), E, between the terminals of R is measured. Then by Ohm's law

$$I = \frac{E}{R}$$

80. The measurement of current by the calorimetric method.—The current, I, to be measured is passed for t seconds through a known resistance R which is submerged in a calorimeter. The rise in temperature is observed, from which the total amount of heat H (in joules) is determined. Then

$$H = RI^2t \qquad (29\ bis)$$

or

$$I = \sqrt{\frac{H}{Rt}}$$

81. Direct reading ammeters.—An ammeter is a galvanometer with a pointer which plays over a calibrated scale and indicates, directly, the value of the current flowing through the instrument. The *tangent-galvanometer type* of ammeter is an instrument in which the pointer is attached to a small suspended magnet which is deflected by the current to be indicated. The *electrodynamometer type* of ammeter is an instrument in which the pointer is attached to the movable coil of an electrodynamometer. In *the D'Arsonval-galvanometer type* of ammeter the pointer is attached to a small pivoted coil of wire which, when the current flows through it, is deflected by a permanent steel magnet. The Weston instruments are of this type.

The *plunger type* of ammeter depends upon the drawing of a soft iron plunger into a coil through which the current flows, the attraction of the coil being counteracted by a spring. A pointer attached to the plunger plays over a calibrated scale. This type of instrument is extensively used where cheapness is a prime consideration.

The *hot wire* type of ammeter depends upon the rise in temperature and consequent expansion of a wire through which the current flows. The expanded wire actuates a pointer which plays over a calibrated scale.

Remark.—The voltmeter, which in most cases is essentially an ammeter, is described in a subsequent article.

Measurement of Resistance.

82. Absolute measurement of resistance. The international standard ohm.—A method for the absolute measurement of resistance was described in Article 46. The most accurate absolute measurements of resistance hitherto carried out are represented in the following specification which was adopted by the International Electrical Congress at Chicago in 1893.

The international standard ohm is the resistance, at the temperature of melting ice, of a column of pure mercury 106.3 centimeters long, of uniform cross-sectional area, and weighing 14.4521 grams.

83. Resistance boxes, or rheostats, are arrangements by means of which any desired resistance may be introduced into a circuit. The usual construction is as follows: A series of massive metal blocks are connected by wires whose resistances are 1, 2, 2, 5, 10, 20, 20, 50 ohms, etc., respectively. The arrangement is shown in Fig. 54. By means of metal plugs which fit snugly between the blocks, the blocks may be connected at pleasure, leaving the resistance between them practically zero.

Fig. 54.

84. Measurement of resistance by the substitution method.—The wire, the resistance of which is to be measured, is connected in circuit with a battery and a galvanometer, and the galvanometer reading is noted. A resistance box is then substituted in place of the wire, and plugs are removed from the box until the galvanometer reading is the same as before. The box reading then gives the value of the unknown resistance.

85. Measurement of resistance by means of the tangent galvanometer.—This method is adapted to the measurement of the resistance of a battery of which the electromotive force does not fall off when it is called upon to give increased current. The battery whose resistance R is to be measured is connected in circuit with a tangent galvanometer and a known resistance a, and the galvanometer deflection ϕ is observed. Then

$$\frac{E}{R+a} = k \tan \phi \qquad \text{(i)}$$

where E is the electromotive force of the battery. The known resistance in circuit is then changed to b and the galvanometer deflection ϕ' is again observed. Then

$$\frac{E}{R+b} = k \tan \phi' \qquad \text{(ii)}$$

Dividing equation (i) by equation (ii) member by member we have

$$\frac{R+b}{R+a} = \frac{\tan \phi}{\tan \phi'} \qquad \text{(iii)}$$

from which R, the only unknown quantity, may be determined.

86. Measurement of resistance by means of the differential galvanometer. —The differential galvanometer consists of a magnet, with attached mirror or pointer, suspended between two similar coils. The distances of these coils from the needle are adjusted until the *same* current flowing in opposite directions in these coils gives no deflection of the suspended magnet. If each of the coils of this galvanometer is connected with a distinct circuit, then when the galvanometer gives no deflection the currents in the two circuits must be equal. The following is the method of using such a galvanometer for the measurement of resistance:

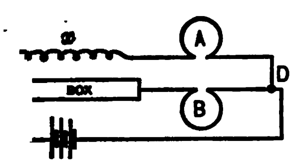


Fig. 55.

battery circuit branches at C and D (Fig. 55). One of these branches in-
s an unknown resistance x and one coil A of a differential galvanometer. The

other branch includes a resistance box and the other coil B of the differential galvanometer. Plugs are removed from the box until the galvanometer gives no deflection. The unknown resistance x is then equal to the box reading.

87. Wheatstone's bridge.—Wheatstone's bridge consists of a network of conductors, as shown in Fig. 56. A battery circuit branches at a and b, and the current flows through resistances α, β, γ and δ, as shown. A sensitive galvanometer G is connected between c and d.

Proposition.—When no current flows through the galvanometer then

$$\frac{\alpha}{\beta}=\frac{\gamma}{\delta} \tag{47}$$

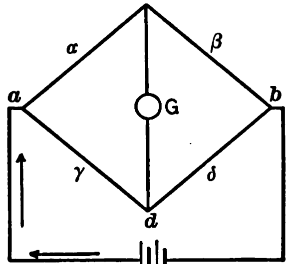


Fig. 56.

Proof.—Let i' be the current flowing through α and β (the same current flows through α and β since the galvanometer current is zero), and let i'' be the current flowing through γ and δ. The electromotive force between c and d is zero, therefore the electromotive force, $\alpha i'$, between a and c is equal to the electromotive force, $\gamma i''$, between a and d; that is

$$\alpha i' = \gamma i'' \tag{i}$$

similarly

$$\beta i' = \delta i'' \tag{ii}$$

Dividing (i) by (ii), member by member, we have $\frac{\alpha}{\beta}=\frac{\gamma}{\delta}$.

Q.E.D.

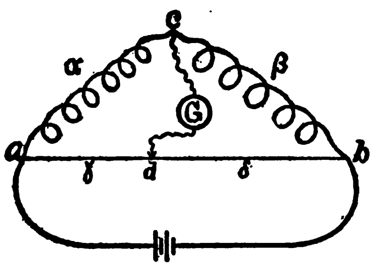


Fig. 57.

The two most important forms of Wheatstone's bridge are the *slide-wire bridge* and the *box bridge*.

88. Measurement of resistance by the slide-wire bridge.—A stretched wire ab (Fig. 57), an unknown resistance α, a known resistance β, and a sensitive galvanometer G are connected as shown. The lettering in Fig. 57 corresponds to that in Fig. 56. The sliding contact d is moved along the wire until the galvanometer gives no deflection. Then $\frac{a}{\beta} = \frac{\gamma}{\delta}$, from equation (47). But $\frac{\gamma}{\delta}$ is equal to the ratio of the lengths of the corresponding portions of the wire ab, and is thus easily determined, so that β being known a may be calculated.

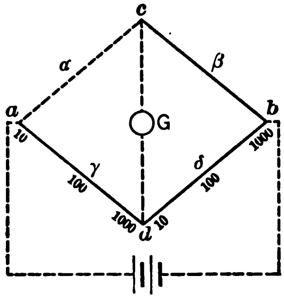


Fig. 58.

89. Measurement of resistance by the box bridge.—The *box bridge* is a resistance box containing three sets of resistances β, γ, and δ, connected as shown in Fig. 58. The dotted lines represent connections outside the box. The portions γ and δ of the box usually have, each, 10-*ohm*, 100-*ohm*, 1,000-*ohm* coils, so that the ratio $\frac{\gamma}{\delta}$ has a number of values which may be chosen at convenience. The portion β contains usually 1, 2, 2, and 5 of each units, tens, hundreds, etc., of ohms. An unknown resistance a is connected as shown, the ratio $\frac{\gamma}{\delta}$ is chosen, and the value of β is changed until the galvanometer gives no deflection. The value of a is then computed from equation (47).

90. The measurement of resistance by the ammeter and voltmeter.—In the testing laboratory, where it is often inconvenient to use Wheatstone's bridge, resistance is most frequently measured by means of an ammeter and a voltmeter (see article 96). A current measured by an ammeter is sent through the resistance to be measured and the electromotive force between the terminals of the resistance is measured by a voltmeter. The resistance is then calculated from Ohm's law, equation (34).

91. Measurement of very high resistances. *Insulation resistance.*—Very large resistances cannot be easily measured by the methods adapted to the measurement of the resistances of wires. These very high resistances may be determined by measuring with a very sensitive galvanometer the current I which is produced in the given resistance by a large electromotive force, E, which is known. Then, according to equation (34), the resistance is equal to $\frac{E}{I}$.*

Example.—One terminal of a storage battery, of which the electromotive force is 1,000 volts, is connected through a very sensitive galvanometer to an outside tin-foil coating on a glass jar, and the other terminal of the battery is connected to an inside coating. The current, indicated by the steady deflection of the galvanometer, is 1.4×10^{-10} amperes. The resistance of the glass between the coatings is therefore 710,000 megohms (one megohm is 1,000,000 ohms).

92. The ohmmeter.—For some purposes it is desirable to use an instrument which gives the resistance of a circuit by direct reading on a calibrated scale. Such an instrument is called an *ohmmeter*. The following discussion is intended only to explain the principle of the ohmmeter. For this purpose the ohmmeter may be considered to

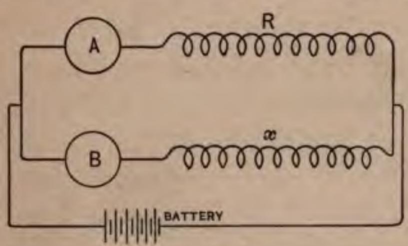


Fig. 59.

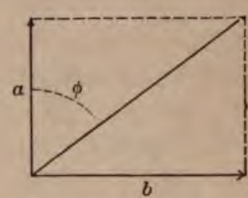


Fig. 60.

be a modified form of tangent galvanometer with two coils A and B, of which the planes are vertical and at right angles to each other, and at the common center of which a magnet needle is suspended. The earth's magnetic field at this needle is neutralized by means of a governing magnet so that the needle is free to point in the direction of the resultant magnetic field due to currents in the two coils. The coils are connected as shown in Fig. 59 in which R represents a fixed resistance attached to

* As a matter of fact, insulators do not conform to Ohm's law or, in other words, the current through an insulator is not strictly proportional to the electromotive force so that different values will be gotten for the insulation resistance according to the value of electromotive force used.

the instrument, and x is the resistance to be measured. Let E be the electromotive force of the battery. Then $\frac{E}{R}$ is the current in coil A and this current produces at the needle a magnetic field a, which is at right angles to the plane of coil A and of which the intensity is proportional to $\frac{E}{R}$. That is,

$$a = k' \frac{E}{R} \quad \text{(i)}$$

The current in coil B is $\frac{E}{x}$ and the magnetic field at the needle due to this current is

$$b = k'' \frac{E}{x} \quad \text{(ii)}$$

The two magnetic fields at the needle are represented by the lines a and b, Fig. 60. When the coil B is open-circuited the magnetic field b is zero and the needle points in the direction of a. When the circuit of coil B is closed the needle turns through the angle ϕ and points in the direction of the resultant field, f. The angle may thus be observed.

Now tan ϕ equals b/a or using equations (i) and (ii) we have

$$\tan \phi = \frac{k''R}{k'x} \quad \text{or} \quad x = \frac{k''R}{k'} \cot \phi$$

This equation shows that a definite value of x corresponds to each value of ϕ and therefore a calibrated scale may be constructed to read ohms directly.

Measurement of Electromotive Force.

93. The international volt. The standard Clark cell.—The international volt is the electromotive force which will produce one international ampere (see Art. 78) through an international ohm (see Art. 82). The international volt is represented sufficiently well for all practical purposes by $\frac{1000}{1434}$ of the electromotive of the Clark cell (see Art. 75) at a temperature of 15° C. The standard specifications * for the Clark cell were given by the International Electrical Congress, Chicago, 1893.

The fundamental method for measuring electromotive force is described in Article 50.

94. Poggendorff's compensation method for measuring electromotive force.—The electromotive force of a voltaic cell, such as a Clark cell, is greatly reduced in value when the cell is called

* See *Transactions* of the American Institute of Electrical Engineers, 1893.

upon to give current. The electromotive force of such a cell cannot, therefore, be accurately measured by the method described in Article 50. This difficulty is avoided by the following compensation or balance method.

A sensitive galvanometer, G, and a voltaic cell, e, of which the electromotive is to be measured, are connected, as shown in Fig. 61, to the terminals of a known resistance r. This resistance is also a portion of a circuit AB through which the current I, produced by some auxiliary electric generator, flows. The current I is adjusted in strength until the galvanometer G gives no deflection. Then the electromotive force (e) of the battery e is equal to the electromotive force produced between the terminals of the resistance r by the current I. Therefore,

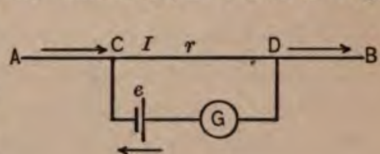


Fig. 61.

$$e = rI$$

The resistance r being known, and the current I being measured by a galvanometer or ammeter placed in the circuit AB, the electromotive force e may be calculated.

95. Measurement of the ratio of two electromotive forces by means of the slide-wire potentiometer.—A constant current, I, is maintained in a stretched wire, AC (Fig. 62), by means of a battery, B. A cell of electromotive force e and a sensitive galvanometer are connected to A, and to a sliding-contact d, which is moved until the galvanometer gives no deflection, then $Ir = e$, as in the previous paragraph. Another cell of electromotive force e' is now put in place of e, and the contact d is again moved until the galvanometer gives no deflection. Then $Ir' = e'$, and we have

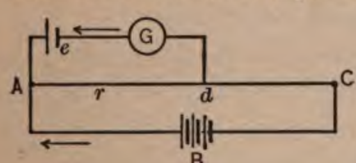


Fig. 62.

$$\frac{e}{e'} = \frac{r}{r'}$$

Since the ratio $\frac{r}{r'}$ is equal to the ratio of the respective lengths of wire between Ad, and is thus easily determined, $\frac{e}{e'}$ is known.

96. Measurement of electromotive force by the galvanometer. Voltmeters.—An electromotive force may be determined, as explained in Article 50, by measuring with a galvanometer the current it produces through a known resistance. A galvanometer of high resistance is adapted to this use. Such a galvanometer is sometimes called a *potential* galvanometer.

The *voltmeter* is a high-resistance ammeter, the scale of which, instead of indicating the current flowing through the instrument, indicates directly the electromotive force between the terminals of the instrument. There is but one type of voltmeter, namely the *electrostatic voltmeter*, which is not essentially an ammeter. This instrument is described in the chapters on electrostatics.

Measurement of Power.

97. Measurement of power by means of the ammeter and voltmeter.—The power delivered to an electrical circuit may be calculated by the equation $P = EI$ (33) when the current I in the circuit and the electromotive force E between the terminals of the circuit have been measured. It is in this way that power expended in electrical circuits is most frequently measured.

98. Determination of power in terms of current and resistance.—When the power delivered to a circuit is all expended in the generation of heat, in accordance with Joule's law, it may be calculated by the equation $P = RI^2$ (30) provided the current I in the circuit has been measured, and the resistance R of the circuit is known.

99. Measurement of power by means of the wattmeter.—The *wattmeter* is an electro-dynamometer, the movable coil of which, A, Fig. 63, carries a pointer which plays over a divided scale. This coil is made of fine wire, and, together with a resistance, R, is connected directly to the mains. The other coil, B, of the electro-dynamometer is connected in series with the circuit L in which the power to be measured is expended. The current in the coil A is $\frac{E}{R}$. The current in the coil B is the current I which flows through the circuit L. The force with which the coil B tends to deflect the coil A is proportional to the product, $\frac{E}{R} \cdot I$, of the currents in the two coils. That is, this force, since R is constant, is proportional to the power, EI, expended in the circuit L. Therefore, for a given power there is a definite force action and a definite deflection and the scale of the instrument may be divided to read watts directly.

Fig. 63.

Measurements of Magnetic Fields.

100. Comparison of strengths of field at two places by the method of vibrations.—A magnet whose moment of inertia is K and whose magnetic moment is M, when suspended at a place at which the horizontal field is H, vibrates in such a way that

$$\frac{4\pi^2 K}{\tau^2} = MH \qquad \text{(i)}$$

in which τ is the period of the vibrations. See equation (7). If the magnet be suspended at a place at which the horizontal field is H_1, we have

$$\frac{4\pi^2 K}{\tau_1^{\,2}} = MH_1 \qquad \text{(ii)}$$

Dividing (i) by (ii), member by member, we have

$$\frac{H}{H_1} = \frac{\tau_1^{\,2}}{\tau^2}$$

so that if τ and τ_1 be observed, $\frac{H}{H_1}$ is known.

101. Comparison of strengths of field at two places by the method of deflections.—If a small magnet be suspended at one of the places and deflected through the angle ϕ by a large magnet, then

$$\tan \phi = \frac{h}{H} \qquad \text{(i)}$$

(See Art. 13.) If the small magnet be now suspended in another place where the field is H_1, and be again deflected through the angle ϕ_1 by means of the same large magnet at the same distance, then

$$\tan \phi_1 = \frac{h}{H_1} \qquad \text{(ii)}$$

Dividing (i) by (ii), member by member, we have

$$\frac{H_1}{H} = \frac{\tan \phi}{\tan \phi_1}$$

so that the ratio $\frac{H'}{H}$ is known when ϕ and ϕ' have been observed.

102. Measurement of field by means of the tangent galvanometer.—Let a current I, measured, for example, by means of a silver voltameter, be sent through a tangent galvanometer, and the deflection ϕ observed; then

$$I = \frac{rH}{2\pi n} \tan \phi \qquad (20) \textit{ bis}$$

If r and n are known, we may measure I and ϕ, and thus obtain data from which H may be calculated.

103. Measurement of strength of field by means of a suspended coil.—Let a current I, measured, for example, by means of a silver voltameter, be sent through a rectangular coil, suspended as explained in Article 36; then

$$T = abnIH \qquad (23) \textit{ bis}$$

is the torque acting upon the coil. If n and r are known and T observed, H may be calculated.

104. Measurement of strength of field by means of the bismuth inductometer.—The bismuth inductometer is a small resistance coil made of fine bismuth wire. Its resistance varies with the intensity of the magnetic field in which it is placed. The relation between resistance and field intensity being once for all determined, the intensity of any field may be found by measuring the resistance of the inductometer when it is placed in the field.

105. Kohlrausch's method for the simultaneous absolute measurement of the horizontal component, *H*, of the earth's magnetic field and of current.—The coil of a tangent galvanometer is suspended so as to enable the measurement of the torque with which H acts upon it. This torque is

$$T = \pi n r^2 I H \qquad (24)\ bis$$

At the same time the deflection ϕ of the needle of the galvanometer is observed. The value of the current is

$$I = \frac{rH}{2\pi n} \tan \phi \qquad (20)\ bis$$

The quantities r and n being known, and T and ϕ being observed, these two equations enable the calculation of both I and H.

CHAPTER VII.

INDUCTANCE.

106. Spark at break.—When an electric circuit is broken the current persists in flowing across the break for a short time, producing, by the heating of the medium through which it passes, an electric arc or spark. This action of the electric current is suggestive of *momentum* and it is convenient, in discussing the phenomena to be treated in this chapter, to speak of the *momentum of the electric current.* The intensity of the spark is a rough indication of the amount of this supposed momentum.

The amount of momentum associated with a *given current* in a circuit made of a *given length and size of wire depends upon the*

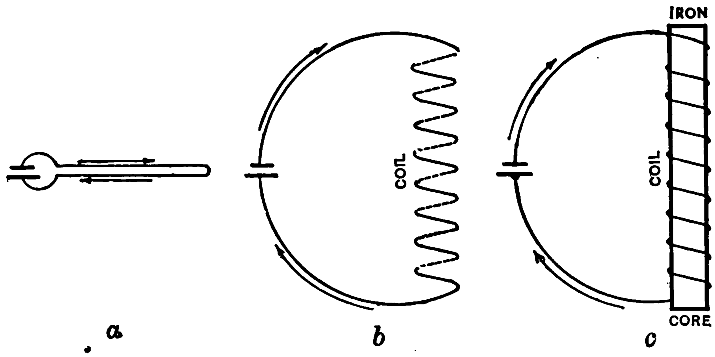


Fig. 64.

shape of the circuit and upon the presence of iron near the circuit. Thus a current in circuit *a*, Fig. 64, has but little momentum; the same current in circuit *b* has greater momentum; and in circuit *c* the same current has very much greater momentum.

When the circuit of an ordinary incandescent lamp is broken a very slight spark only is produced. The same amount of current flowing through a coil of wire produces a much greater spark when the circuit is broken, and a spark several inches in length may be produced if the coil of wire surrounds an iron core consisting of a bundle of iron wires.

The magnetic field which is produced by an electric current seems to be a state of motion of the ether and the momentum of the electric current depends upon this magnetic field. Thus a current in the circuit *a*, Fig. 64, produces a very weak magnetic field, except in the small region near the wires, and the momentum of the current is small. The same current in circuit *b* produces an intense magnetic field inside of the coil and the momentum of the current is correspondingly great. The magnetism of the iron core in circuit *c* accounts for the very great momentum of the current in that circuit.

In the following discussion the notion of *kinetic energy* is used in preference to the notion of *momentum* for the reason, mainly, that kinetic energy is expressed in mechanical units even when it is electrical kinetic energy.

107. The kinetic energy associated with a current in a circuit. Definition of inductance.—The ether motion which constitutes magnetic field represents kinetic energy and *the kinetic energy associated with an electric current is the total energy residing in the magnetic field produced by the current.* We shall for convenience call this the kinetic energy of the current.

The amount of energy residing in a portion of a magnetic field is proportional to the square of the intensity of that portion of the field. This is analogous to the fact that the energy of a portion of a moving liquid is proportional to the square of the velocity of that portion of the liquid. If the current in a circuit is doubled, the field intensity is everywhere doubled, so that the energy of each portion of the field is quadrupled. Therefore the total energy of the field is quadrupled when the current is doubled or *the kinetic energy of a current is proportional to the square of the current.* That is

$$W = \tfrac{1}{2}Li^2 \tag{48}$$

in which W is the kinetic energy of a current i in a given circuit, and $(\frac{1}{2}L)$ is the proportionality factor. The quantity L is called the *inductance* of the circuit. The inductance of a circuit is essentially a positive quantity inasmuch as the kinetic energy W in equation (48) cannot be negative.

Moment of inertia of a wheel. Analogue of inductance.—The kinetic energy of a rotating wheel resides in the various moving particles of the wheel. If the angular velocity of the wheel is doubled, the linear velocity of every particle of the wheel is doubled, so that the energy of every particle is quadrupled. Therefore the total energy is quadrupled when the angular velocity is doubled, so that *the total energy is proportional to the square of the angular velocity*. That is

$$W = \tfrac{1}{2}K\omega^2 \tag{49}$$

in which W is the kinetic energy of a rotating wheel, ω is the angular velocity of the wheel, and $(\frac{1}{2}K)$ is the proportionality factor. The quantity K is called the *moment of inertia* of the wheel.

Units of inductance.—If W in equation (48) is expressed in ergs and i in c.g.s. units, L is expressed in c.g.s. units of inductance. This c.g.s. unit of inductance is called the *centimeter*, for the reason that the square of a current must be multiplied by a length to give energy or work; that is, inductance is expressed as a length, and the unit of inductance is, of course, the unit of length. If W in equation (48) is expressed in joules and i in amperes, then L is expressed in terms of a unit called the *henry*. The henry is equal to 10^9 centimeters of inductance.

Inductance of a coil.—Strictly, one cannot speak of the inductance of anything but an *entire circuit*, inasmuch as every portion of a circuit contributes its share to the magnetic field at each and every point; it is, however, allowable to speak of the inductance of a coil when the terminals of the coil are not too far apart and when the *remainder* of the electric circuit does not produce any perceptible magnetic field in the region occupied by the coil.*

* See Art. 118.

When the inductance of a coil is so small that it is negligible the coil is said to be noninductive.

Measurement of inductance.—The most accurate method for determining the inductance of a coil is by calculation from measured dimensions. This calculation can be carried out only when the coil is simple in shape and even then the calculation is in most cases quite complicated. The simplest case is given in Article 113. The inductance of an irregularly shaped coil may be determined by various electrical methods. See A. Gray's *Absolute Measurements*, Vol. II., Part II., pp. 438–509.

108. Electromotive force required to increase or decrease a current.—A current once established in a circuit of *zero resistance* would continue to flow without the help of an electromotive force to maintain it, just as a wheel once started would continue to turn without the help of a driving torque if there were no resistance to the motion of the wheel. To increase the speed of the wheel a torque must act upon it in the direction of its rotation, and to increase the current in a circuit an electromotive force must act on the circuit in the direction of the current; also to reduce the speed of the wheel an opposing torque must act upon it, and to decrease the current in a circuit an opposing electromotive force must act upon the circuit.

To maintain a constant current in a circuit having resistance a definite electromotive force is needed and if an electromotive force e, over and above the electromotive force required to overcome the resistance of the circuit, acts upon a circuit the current is made to increase at a definite *rate* such that

$$e = L\frac{di}{dt} \tag{50}$$

in which L is the inductance of the circuit, and $\frac{di}{dt}$ is the rate of increase of the current. When e is opposed to the current it is considered negative and $\frac{di}{dt}$ is negative, that is, the current decreases.

Proof of equation (50).—Multiplying both members of this equation by the current i, we have $ei = Li\frac{di}{dt}$. Now ei is the rate, $\frac{dW}{dt}$, at which work is done on the circuit, according to equation (33), over and above the work required to overcome resistance, and this must be equal to the rate at which the kinetic energy of the current is increasing. Differentiating equation (48) we have $\frac{dW}{dt} = Li\frac{di}{dt}$.

Further development of the analogy between the moment of inertia of a wheel and the inductance of a circuit.—When a torque T, over and above the torque required to overcome frictional resistance, acts upon a rotating wheel the angular velocity of the wheel is made to increase at a definite *rate* such that

$$T = K\frac{d\omega}{dt} \tag{51}$$

in which K is the moment of inertia of the wheel, and $\frac{d\omega}{dt}$ is the rate of increase of the speed.

Proof of equation (51).—Multiply both members of this equation by the angular velocity ω, we have $T\omega = K\omega\frac{d\omega}{dt}$. Now $T\omega$ is the rate, $\frac{dW}{dt}$, at which work is done on the wheel over and above the work required to overcome resistance, and this must be equal to the rate at which the kinetic energy of the wheel is increasing. Differentiating equation (49) we have $\frac{dW}{dt} = K\omega\frac{d\omega}{dt}$.

109. Self-induced electromotive force. Reaction of a changing current.—When one pushes on a wheel, causing its speed to increase, the wheel reacts and pushes back against the hand. This reacting torque is equal and opposite to the acting torque, $K\frac{d\omega}{dt}$, which is causing the increase of speed. When the speed of a wheel is increasing the reaction of the wheel is a torque opposed to its motion; when the speed is decreasing the reaction is a torque in the direction of the motion.

Similarly, when an electromotive force acts upon a circuit and causes the current to decrease or increase, the changing current reacts. This reacting electromotive force is equal and opposite to the acting electromotive force, $L\frac{di}{dt}$, which is causing the current to change. When the current is increasing the reaction of the current is an electromotive force opposed to the current, when the current is decreasing the reaction of the current is an electromotive force in the direction of the current. The *reaction* of a changing current is called *self-induced* electromotive force. The self-induced electromotive force in a circuit is,

$$e = -L\frac{di}{dt} \tag{52}$$

Remark.—When a rotating wheel is left to itself without any outside torque to maintain its motion, the motion dies down on account of resistance or friction. The torque which overcomes the resistance is the reaction of the wheel.

Similarly, when a current in a circuit is left to itself without any electromotive force to maintain it, it dies down on account of resistance. The electromotive force, Ri,* which overcomes the resistance is the reaction of the decreasing current.

Consider an electromotive force E, due to a battery or dynamo, acting upon a circuit having a resistance R and an inductance L. Part of the electromotive force is used to overcome resistance. This part is equal to Ri. Part of the electromotive force is used to make the current increase or decrease. This part is equal to $L\frac{di}{dt}$. The total electromotive force is the sum of these two parts so that

$$E = Ri + L\frac{di}{dt} \tag{53}$$

Examples.—(*a*) An electric generator having an electromotive force of 110 volts is, at a given instant, connected to a circuit of which the resistance is 3 ohms and the inductance is 0.04 henry. The curve of growing current is shown in Fig. 65, time being

* Whatever other actions and reactions there may be in a circuit, an electromotive force equal to Ri is required to overcome the resistance of the circuit, according to equation (34).

reckoned from the instant that the generator is connected to the circuit. At this instant the current i is zero, so that, according to equation (53), $L\frac{di}{dt}$ is equal to 110 volts, and $\frac{di}{dt}$ is equal to 2750 amperes per second.

After 0.02 second the value of the growing current is 28.5

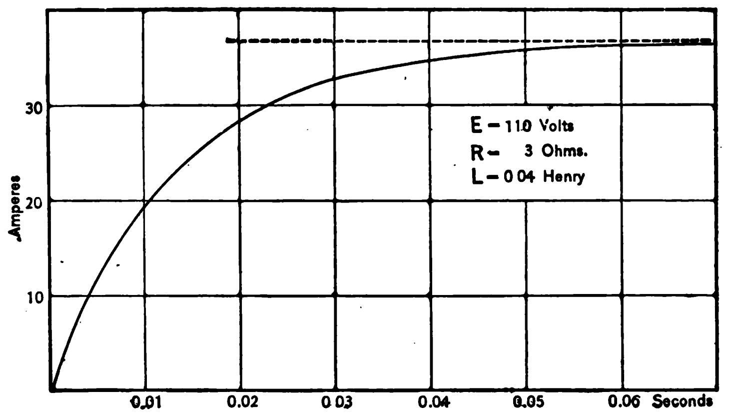


Fig. 65.

amperes, so that Ri equals 85.5 volts, $L\frac{di}{dt}$ equals 24.5 volts, and $\frac{di}{dt}$ equals 612.5 amperes per second.

After a few tenths of a second the growing current reaches very nearly its full value. Then $\frac{di}{dt}$ is zero, Ri equals 110 volts, and i equals 36.7 amperes.

(*b*) The circuit above described has a current of 50 amperes established in it and the circuit is then left to itself without any electromotive force acting upon it to maintain the current. The curve of decaying current is shown in Fig. 66, time being reckoned from the instant that the decaying current has the value of 36.7 amperes. At this instant Ri equals 110 volts so that $L\frac{di}{dt}$ equals ·olts, and $\frac{di}{dt}$ equals -2750 amperes per second.

After 0.02 second the value of the decaying current is 8.2 amperes. Then Ri equals 24.6 volts, $L\frac{di}{dt}$ equals -24.6 volts, and $\frac{di}{dt}$ equals -615 amperes per second.

The equation of the curve of growing current is

$$i = \frac{E}{R} - \frac{E}{R}\varepsilon^{-\frac{R}{L}.t} \tag{54}$$

in which ε is the Naperian base and i is the value of the growing current t seconds after the electromotive force E is connected to the circuit.

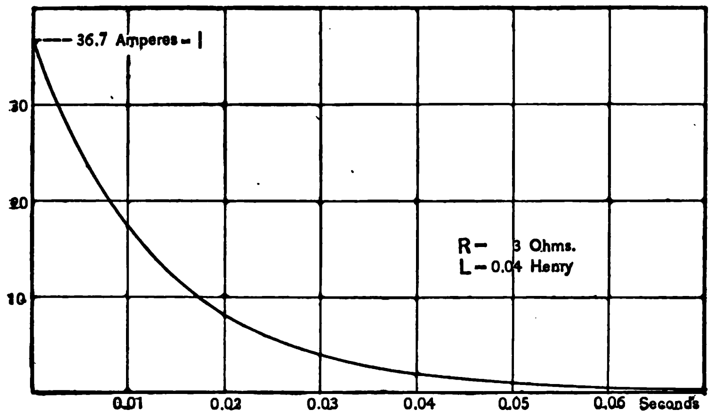


Fig. 66.

The equation of the curve of decaying current is

$$i = I\varepsilon^{-\frac{R}{L}.t} \tag{55}$$

in which, as before, ε is the Naperian base, I is the value of the decaying current at the instant from which time is reckoned, and i is the value of decaying current t seconds later.

110. The choke coil.—A coil having considerable inductance is frequently used for the choking of rapid fluctuations of current. Such a coil is called a *choke coil.* When such a coil is connected to the terminals of an alternator the rapidly alternating electromotive force of the alternator produces but little current through the coil just as a rapidly alternating torque produces but little to and fro motion of a massive wheel. The choke coil is used in connection with the lightning arrester as follows:

Fig. 67 represents a dynamo, D, supplying current to a trolley wire. When this wire is struck by lightning a sudden rush of current takes place through D to earth. This rush of current may prove disastrous to the dynamo which is, therefore, protected as follows: A coil of wire, C, the *choke coil*, is connected as shown in the figure. The sudden establishment, in this coil, of the current which follows a lightning stroke requires, as will

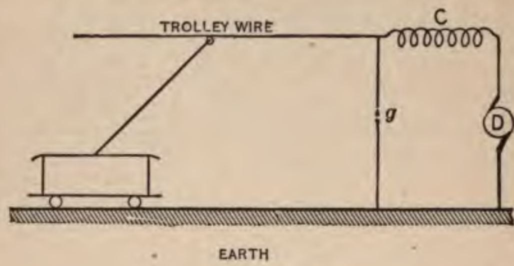


Fig. 67.

be seen by reference to equation (50), a very great electromotive force. This electromotive force produces a spark across a narrow air gap, g. This spark makes the air gap a comparatively good conductor and the greater portion of the rush of current passes harmlessly to earth through g. Such an arrangement is called a *lightning arrester*. A lightning arrester must be provided with an arrangement for stopping the flow of the dynamo current across the gap g after the rush of current from the lightning stroke has ceased. This is sometimes done, as in the Thomson arrester, by means of a strong magnet which produces an intense magnetic field in the region of the gap g, thus pushing the electric arc sidewise and blowing it out. This is called the *magnetic blow-out*.

Example.—A coil of heavy copper wire has its terminals one centimeter apart. One of these terminals is connected to ground, and the other terminal is connected to a metal rod which is struck by a spark from an electric machine. The sudden rush of current, due to the spark, jumps across between the terminals of the coil instead of following along the heavy copper wire, although the electrical resistance of the heavy wire is very small. A coil consisting of 100 turns of wire wound on a wooden cylinder 4 centimeters in diameter, has approximately 0.00005 henry of inductance. At the instant that the spark jumps across between the terminals of this coil the electromotive force between the terminals must be about 20,000 volts, which is the electromotive force required to make a spark one centimeter long. Therefore, neglecting the resistance of the coil, the whole of this 20,000 volts must have been causing the current to increase, so that 20,000 volts $= 0.00005 \times \frac{di}{dt}$, or $\frac{di}{dt} = 400{,}000{,}000$ amperes per second.

111. Magnetic flux through a coil due to a current in the coil.—The *magnetic flux through a coil* is the product of the flux through a mean turn multiplied by the number of turns of wire in the coil. That is, the magnetic flux through the mean turn is counted as many times as there are turns of wire. Let Φ_1 be the flux through the opening of a coil (through a mean turn), Φ the flux through the coil and Z the number of turns of wire in the coil, then

$$\Phi = Z\Phi_1 \tag{56}$$

The flux through a coil due to a current in the coil is equal to the product of the inductance of the coil into the current. That is,

$$\Phi = Li \tag{57}$$

Proof.—The self-induced electromotive force in a coil may be looked upon as due to the changing flux which accompanies changing current. Then, according to equation (40), the self-induced electromotive force would be equal to $-\frac{d\Phi}{dt}$. Placing this equal to $-L\frac{di}{dt}$, which, according to equation (52), is equal to the self-induced electromotive force, we have $\frac{d\Phi}{dt} = L\frac{di}{dt}$, whence, integrating from $i = 0$ and $\Phi = 0$, we have $\Phi = Li$.

This simple case of integration occurs once or twice in subsequent chapters.

The operation, plainly stated, is as follows: The equation $\frac{d\Phi}{dt} = L\frac{di}{dt}$ means that Φ *increases L times as fast as i.* Therefore *if Φ and i start from zero together then Φ is always L times as large as i. That is, $\Phi = Li$.*

112. The dependence of the inductance of a coil upon the number of turns of wire in the coil and upon the size of the coil.*— *The inductance of a coil of wire* **wound on a given spool** *is proportional to the square of the number of turns of wire.* Thus, a given spool wound with No. 16 wire has 500 turns of wire and an inductance of, say, 0.025 henry; the same spool wound full of No. 28 wire would have about ten times as many turns and its inductance would be about 100 times as great, or about 2.5 henrys.

Proof.—To double the number of turns of wire on a given spool would everywhere double the magnetic field intensity *for the same current.* Therefore the energy of the field would be quadrupled, as explained in Article 104, so that the inductance would be quadrupled according to equation (48).

The inductance of a coil of given shape, the number of turns of wire being unchanged, is proportional to its linear dimensions.

113. Calculation of inductance in terms of magnetic flux per unit current.—According to equation (57) the inductance of a coil is equal to the quotient $\frac{\Phi}{i}$, where Φ is the *magnetic flux through the coil* due to current i in the coil. There are important cases in which the flux through a coil due to a given current may be easily calculated so that the inductance of such a coil is easily determined.

Inductance of a long coil.—Wire is wound in a thin layer on a long tube, z turns of wire per unit length of tube. The field intensity inside this coil, according to Article 122, is $f = 4\pi zi$. Let q be the area of the opening of the coil, then $qf = 4\pi qzi (= \Phi_1)$ is the flux through the opening. Let l be the length of the tube, the total number of turns is then lz and the *flux through the coil*

* The dependence of inductance upon the *shape* of a coil is much too complicated to permit of its general discussion in this text. The next article gives a discussion of the simplest case.

is $4\pi q z^2 l i$ which divided by i gives the inductance of the coil, so that

$$L = 4\pi q z^2 l \tag{58}$$

This equation is strictly true only for very long coils with thin* windings of wire. For short coils the values of L given by this equation are too great. However, this equation is useful in the calculation of the approximate inductance of even a short coil.

114. Kinetic energy associated with independent currents in two circuits. Definition of mutual inductance.—Consider two adjacent circuits, one of which may be called the *primary* and the other the *secondary* to distinguish them. Let $i_{\prime}$ be the current in the primary circuit, and $i_{\prime\prime}$ the current in the secondary circuit. The total energy associated with these two currents consists of three parts: (a) a part which is proportional to $i_{\prime}^2$, (b) a part which is proportional to $i_{\prime\prime}^2$, and (c) a part which is proportional to $i_{\prime} i_{\prime\prime}$. Therefore we may write:

$$W = \tfrac{1}{2} L_{\prime} i_{\prime}^2 + \tfrac{1}{2} L_{\prime\prime} i_{\prime\prime}^2 + M i_{\prime} i_{\prime\prime} \tag{59}$$

in which W is the total energy of the two currents, and $(\frac{1}{2}L_{\prime})$, $(\frac{1}{2}L_{\prime\prime})$, and M are the proportionality factors. The quantities $L_{\prime}$ and $L_{\prime\prime}$ are the inductances of the respective circuits, inasmuch as equation (59) reduces to equation (48) when either current is zero. The quantity M is called the *mutual inductance* of the two circuits. It may be either positive or negative. Mutual inductance is expressed in terms of the same units as inductance.

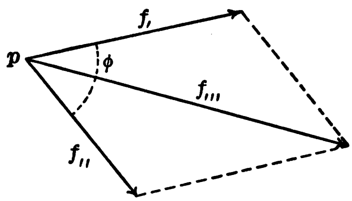


Fig. 68.

Proof of equation (59).—Consider a given point p in the neighborhood of the two circuits. Let $f_{\prime}$, Fig. 68, be the intensity at p of the magnetic field due to $i_{\prime}$ alone, and let $f_{\prime\prime}$ be the intensity at p of the magnetic field due to $i_{\prime\prime}$ alone. The resultant intensity is $f_{\prime\prime\prime}$ as shown in the figure, and we have:

$$f_{\prime\prime\prime}^2 = f_{\prime}^2 + f_{\prime\prime}^2 + 2 f_{\prime} f_{\prime\prime} \cos\phi \tag{i}$$

Now, $f_{\prime} = a i_{\prime}$ (that is, $f_{\prime}$ is proportional to $i_{\prime}$) and $f_{\prime\prime} = b i_{\prime\prime}$ so that equation (i) becomes:

$$f_{\prime\prime\prime}^2 = a^2 i_{\prime}^2 + b^2 i_{\prime\prime}^2 + c i_{\prime} i_{\prime\prime} \tag{ii}$$

in which c is written for $2ab\cos\phi$. The kinetic energy, ΔW, in a volume element at the point p is proportional to $f_{\prime\prime\prime}^2$ or equal, say, to $k f_{\prime\prime\prime}^2$, so that

$$\Delta W = a^2 k \cdot i_{\prime}^2 + b^2 k \cdot i_{\prime\prime}^2 + ck \cdot i_{\prime} i_{\prime\prime} \tag{iii}$$

* If the winding is deep then

$$L = \tfrac{2}{3}\pi^2 z^2 l (R^4 - 6R^2 r^2 + 8Rr^3 - 3r^4)$$

in which z is the number of turns of wire per unit length, l is the length, R is the outside radius, and r is the inside radius of the coil.

That is, this energy consists of three parts proportional respectively to $i_{\prime}^2$, to $i_{\prime\prime}^2$, and to $i_{\prime}i_{\prime\prime}$, therefore, the total energy consists of three such parts. Q.E.D.

Remark.—Mutual inductance is expressed in *centimeters* or *henrys*, the same as inductance.

115. Electromotive forces required to increase or diminish the currents in two circuits.—Let it be required to find the electromotive force $e_{\prime}$ which must act on the primary circuit, and the electromotive force $e_{\prime\prime}$ which must act on the secondary circuit *in order that $i_{\prime}$ may be made to change at the rate $\frac{di_{\prime}}{dt}$, and $i_{\prime\prime}$ be kept constant.*

The rate at which work is done on the primary circuit is $e_{\prime}i_{\prime}$ and the rate at which work is done on the secondary circuit is $e_{\prime\prime}i_{\prime\prime}$, so that the total rate at which work is done on the system is

$$\frac{dW}{dt} = e_{\prime}i_{\prime} + e_{\prime\prime}i_{\prime\prime} \tag{a}$$

From equation (59), under the prescribed conditions, namely, that $i_{\prime\prime}$ is constant and $i_{\prime}$ varying, we have for the rate of increase of the energy of the system:

$$\frac{dW}{dt} = L_{\prime}i_{\prime}\frac{di_{\prime}}{dt} + Mi_{\prime\prime}\frac{di_{\prime}}{dt} \tag{b}$$

Now the right-hand members of equations (a) and (b) are identically* equal, so that:

$$e_{\prime} = L_{\prime}\frac{di_{\prime}}{dt} \tag{c}$$

and

$$e_{\prime\prime} = M\frac{di_{\prime}}{dt} \tag{d}$$

The *reactions* of the two currents are equal and opposite to the electromotive forces $e_{\prime}$ and $e_{\prime\prime}$ which are causing the currents to change in the prescribed manner. Therefore:

$$e_{\prime} = -L_{\prime}\frac{di_{\prime}}{dt} \tag{52) bis}$$

and

$$e_{\prime\prime} = -M\frac{di_{\prime}}{dt} \tag{60}$$

ad $e_{\prime\prime}$ are the reactions in primary and secondary circuit, respectively, due g primary current. The electromotive force $e_{\prime}$ is the self-induced electro-e which is described in Article 109, and *$e_{\prime\prime}$ is an electromotive force in-secondary circuit by the changing primary current.*

gnetic flux through the secondary circuit due to a current in the .—A current in the primary circuit produces a magnetic field in the sur-region and some of the lines of force of this field pass through the sec-

sistance of each circuit is supposed to be zero for the sake of simplicity of that all the work done on the system goes to increase its kinetic energy.

ondary circuit. Let $\Phi_{\prime\prime}$ be the magnetic flux through the secondary circuit due to a primary current $i_{\prime}$. Then

$$\Phi_{\prime\prime} = Mi_{\prime} \tag{61}$$

Proof.—The electromotive force $e_{\prime\prime}$ induced in the secondary circuit by the changing primary current is equal to $-\frac{d\Phi_{\prime\prime}}{dt}$ according to equation (40). Therefore, from equation (60), we have

$$\frac{d\Phi_{\prime\prime}}{dt} = M\frac{di_{\prime}}{dt}$$

or, integrating from $i_{\prime} = 0$ and $\Phi_{\prime\prime} = 0$, we have

$$\Phi_{\prime\prime} = Mi_{\prime} \qquad \text{Q. E. D.}$$

117. The force action between two circuits. *Preliminary statement.*—The work ΔW expended on a circuit to keep a current i in that circuit constant while the magnetic flux through the circuit changes by the amount $\Delta\Phi$, is

$$\Delta W = i \cdot \Delta\Phi \tag{a}$$

This proposition is proved in Article 124.

Let it be required to find the work expended on two circuits when one of them is displaced by an amount Δx, the other circuit being fixed and the currents $i_{\prime}$ and $i_{\prime\prime}$ being kept constant. Let ΔM be the change in M due to the displacement. This increase in M, other things being constant, necessitates an increase of the kinetic energy of the system which, according to equation (59), is

$$u = \Delta M \cdot i_{\prime}i_{\prime\prime} \tag{b}$$

From equation (61) it is evident that the change in M produces a change in the flux through each circuit. In fact $\Delta\Phi_{\prime\prime} = \Delta M \cdot i_{\prime}$ and $\Delta\Phi_{\prime} = \Delta M \cdot i_{\prime\prime}$. Therefore, from equation (a):

$$v = \Delta M \cdot i_{\prime}i_{\prime\prime} \tag{c}$$

is the work spent on the primary circuit to keep $i_{\prime}$ constant during the displacement and

$$w = \Delta M \cdot i_{\prime}i_{\prime\prime} \tag{d}$$

is the work spent on the secondary circuit to keep $i_{\prime\prime}$ constant during this displacement. The total work spent on the system by equations (c) and (d) is therefore $2\Delta M \cdot i_{\prime}i_{\prime\prime}$ while the increase of the energy of the system by equation (b) is only $\Delta M \cdot i_{\prime}i_{\prime\prime}$. Therefore the displacement Δx must have been helped by a force X such that the work done by this force, namely $X \cdot \Delta x$, may be equal to $2\,\Delta M \cdot i_{\prime}i_{\prime\prime} - \Delta M \cdot i_{\prime}i_{\prime\prime}$. That is, $X \cdot \Delta x = \Delta M . i_{\prime}i_{\prime\prime}$ or

$$X = \frac{dM}{dx} \cdot i_{\prime}i_{\prime\prime} \tag{62}$$

This force tends to increase M when the product $i_{\prime}i_{\prime\prime}$ is positive. If Δx is an angular displacement about a given axis then X is a torque acting about that axis.

118. The inductance of two coils connected in series.—Let $L_{\prime}$ be the inductance of one circuit, $L_{\prime\prime}$ the inductance of another circuit, and M their mutual inductance. The total energy of the two circuits is :

$$W = \tfrac{1}{2}L_{\prime}i_{\prime}^2 + \tfrac{1}{2}L_{\prime\prime}i_{\prime\prime}^2 + Mi_{\prime}i_{\prime\prime} \qquad (59)\ bis$$

If these two circuits are connected in series then $i_{\prime} = i_{\prime\prime}$ and we have

$$W = (\tfrac{1}{2}L_{\prime} + \tfrac{1}{2}L_{\prime\prime} + M)i^2$$

or

$$W = \tfrac{1}{2}(L_{\prime} + L_{\prime\prime} + 2M)i^2$$

Therefore the inductance of two coils connected in series exceeds the sum of their individual inductances by the amount $2M$, which may be positive or negative. *A given portion of a circuit, a coil for example, may be treated by itself, or may be treated as having a definite inductance independent of the remainder of the circuit, only when the mutual inductance of the coil and the remainder of the circuit is zero.*

119. Calculation of mutual inductance in terms of flux per unit current.—According to equation (61) the mutual inductance of two coils is equal to the quotient $\frac{\Phi_{\prime\prime}}{i_{\prime}}\left(\text{or } \frac{\Phi_{\prime}}{i_{\prime\prime}}\right)$ where $\Phi_{\prime\prime}$ is the magnetic flux through the one coil due to a current $i_{\prime}$ in the other coil. There are important cases in which the flux through a coil due to a given current in another coil may be easily calculated so that the mutual inductance in such a case is readily determined.

Mutual inductance of a large circular coil (*primary*) *and a small coil* (*secondary*) *at its center.*—Let $r_{\prime}$ be the radius of the large coil and $z_{\prime}$ the number of turns of wire in the coil. Let $r_{\prime\prime}$ be the radius of the small coil and $z_{\prime\prime}$ the number of turns of wire in this coil ; and let θ be the angle between the planes of the coils.

The magnetic field at the center of the large coil is perpendicular to the plane of that coil and its intensity is $f = \frac{2\pi z_{\prime} i_{\prime}}{r_{\prime}}$, from equation (17). The projection, upon the plane of the large coil, of the effective area of the small coil is $\pi r_{\prime\prime}^2 z_{\prime\prime} \cos\theta$, so that the magnetic flux $\Phi_{\prime\prime}$ through the small coil is $\frac{2\pi z_{\prime} i_{\prime}}{r_{\prime}} \times \pi r_{\prime\prime}^2 z_{\prime\prime} \cos\theta$. Dividing this flux by $i_{\prime}$ we have

$$M = \frac{2\pi^2 r_{\prime\prime}^2 z_{\prime} z_{\prime\prime} \cos\theta}{r_{\prime}} \qquad (63)$$

It is of interest to derive the expression for the torque T which acts upon the small coil, tending to change the angle θ. This expression results at once from the substitution of the value of M from equa-ion (63) in equation (62), remembering that the differentiation s with respect to θ. This gives

$$T = -\frac{2\pi^2 r_{\prime\prime}^2 z_{\prime} z_{\prime\prime} \sin\theta}{r_{\prime}} \cdot i_{\prime} i_{\prime\prime} \qquad (64)$$

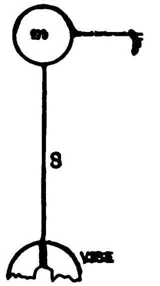
mutual inductance of a long solenoid (*primary*) *and a* *il* (*secondary*) *surrounding it.*—Let $r_{\prime}$ be the mean f the long coil (see Fig. 69) and let $z_{\prime}$ be the number of f wire per centimeter length of coil. The intensity of gnetic field inside of the long coil is $4\pi z_{\prime} i_{\prime}$ according to equation (66). This, multiplied by the area of the opening of the long coil ($\pi r_{,}^2$), gives $4\pi^2 r_{,}^2 z_{,} i_{,}$ as the flux through the opening. This flux also passes through the $z_{,,}$ turns of the secondary coil so that $\Phi_{,,} = z_{,,} \times 4\pi^2 r_{,}^2 z_{,} i_{,}$, whence, dividing by $i_{,}$, we have

$$M = 4\pi^2 r_{,}^2 z_{,} z_{,,} \qquad (65)$$

Mechanical and Electrical Analogies.

120. Mechanical and electrical analogies.—The analogy between moment of inertia and inductance as pointed out in the discussion of inductance is but a small part of an extended analogy between pure mechanics and electricity. This extended analogy is here briefly outlined.

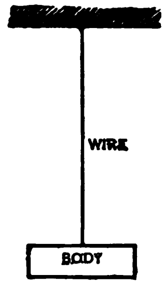
$x = vt \qquad (1)$	$\phi = \omega t \qquad (2)$	$q = it \qquad (3)$
in which x is the distance traveled in t seconds by a body moving at velocity v.	in which ϕ is the angle turned in t seconds by a body turning at angular velocity ω.	in which q is the electric charge which in t seconds flows through a circuit carrying a current i.
$W = Fx \qquad (4)$	$W = T\phi \qquad (5)$	$W = Eq \qquad (6)$
in which W is the work done by a force F in pulling a body through the distance x.	in which W is the work done by a torque T in turning a body through the angle ϕ.	in which W is the work done by an electromotive force E in pushing a charge q through a circuit.
$P = Fv \qquad (7)$	$P = T\omega \qquad (8)$	$P = Ei \qquad (9)$
in which P is the power developed by a force F acting upon a body moving at velocity v.	in which P is the power developed by a torque T acting on a body turning at angular velocity ω.	in which P is the power developed by an electromotive force E in pushing a current i through a circuit.
$W = \frac{1}{2}mv^2 \qquad (10)$	$W = \frac{1}{2}K\omega^2 \qquad (11)$	$W = \frac{1}{2}Li^2 \qquad (12)$
in which W is the kinetic energy of a mass m moving at velocity v.	in which W is the kinetic energy of a wheel of moment of inertia K turning at angular velocity ω.	in which W is the kinetic energy of a coil of inductance L carrying a current i.
$F = m\frac{dv}{dt} \qquad (13)$	$T = K\frac{d\omega}{dt} \qquad (14)$	$E = L\frac{di}{dt} \qquad (15)$
in which F is the force required to cause the velocity of a body of mass m to increase at the rate $\frac{dv}{dt}$	in which T is the torque required to cause the angular velocity of a wheel of moment of inertia K to increase at the rate $\frac{d\omega}{dt}$	in which E is the electromotive force required to cause a current in a coil of inductance L to increase at the rate $\frac{di}{dt}$
$x = aF \qquad (16)$	$\phi = bT \qquad (17)$	$q = CE \qquad (18)$
$\frac{4\pi^2 m}{\tau^2} = \frac{1}{a} \qquad (19)$	$\frac{4\pi^2 K}{\tau^2} = \frac{1}{b} \qquad (20)$	$\frac{4\pi^2 L}{\tau^2} = \frac{1}{C} \qquad (21)$



a

Fig. a.

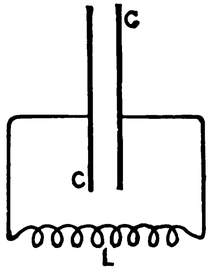
A body of mass m is supported by a flat spring S clamped in a vise as shown in Fig. *a*. A force F pushing sidewise on m moves it a distance x which is proportional to F according to equation (16). When started the body m will continue to vibrate back and forth and the period τ of its vibrations is determined by equation (19).



b

Fig. b.

A body of moment of inertia K is hung by a wire as shown in Fig. *b*. A torque T acting on the body will turn the body and twist the wire through an angle ϕ which is proportional to T according to equation (17). When started, the body will vibrate about the wire as an axis and the period τ of its vibrations is determined by equation (20).



c

Fig. c.

A condenser of capacity C is connected to the terminals of a coil of inductance L as shown in Fig. *c*. An electromotive force E acting anywhere in the circuit pushes into the condenser a charge q which is proportional to E according to equation (18). When started the electric charge will surge back and forth through the coil constituting what is called an oscillatory current and the period of one oscillation is determined by equation (21).

CHAPTER VIII.

MAGNETISM OF IRON.

121. Preliminary statement.—Many practical appliances, such as the dynamo and the transformer, depend for their action upon the magnetization of a rod, or core, of iron. This magnetization is usually produced by an electric current flowing through a winding of wire which surrounds the iron to be magnetized, as described in Art. 19. Two distinct cases arise in the arrangement of these windings as follows:

a. The case in which the wire is wound uniformly on a long iron rod. In this case the intensity of the magnetic field produced by the winding is the same throughout the iron core. This case seldom occurs in practice except in the magnetic testing of iron.

b. The case in which the winding of wire is bunched at certain places along the iron core. In this case the intensity of the magnetic field produced by the winding is not the same throughout the iron core. This case usually occurs in practice.

122. The magnetizing action of a long uniformly wound coil depends simply upon the intensity of the magnetic field inside of the coil and it is important to know the relation between the intensity of this field and the strength of the current in the coil. This relation is:

$$H = 4\pi z i \qquad (66)$$

in which H is the field intensity, in c.g.s. units, inside of a long coil, z is the number of turns of wire per centimeter length of the coil, and i is the current in the coil in c.g.s. units.

When the current i is expressed in amperes we have:

$$H = \frac{4\pi}{10} \cdot zi_{\text{amp.}} \tag{67}$$

Proof.—Equation (66) gives the field intensity at every point inside of a long coil, no matter what the shape of the section of the coil may be. The following proof, however, applies only to the region along the axis of a cylindrical coil. The complete proof is too indirect and too complicated to give here.

Let AB, Fig. 70, be the long coil, of radius r, having z turns of wire per centimeter length, and carrying current i. Let p be a point, in the axis of the coil, at which the field intensity is to be determined. Each element of the coil contributes its

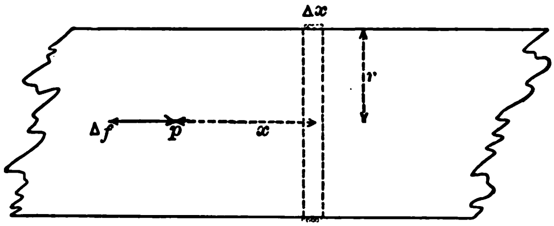


Fig. 70.

share to the field at p. Let Δf be the field intensity at p due to that element of the coil which is at a distance x to the right of p, which element is of length Δx, and has $z \cdot \Delta x$ turns of wire. Then from equation (18) we have

$$\Delta f = \frac{2\pi i r^2 z \cdot \Delta x}{(r^2 + x^2)^{\frac{3}{2}}}$$

Integrating this expression from $x = +\infty$ to $x = -\infty$ we have

$$f = 4\pi z i \qquad \text{Q. E. D.}$$

123. The magnetizing action of a bunched winding. Definition of magnetomotive force.—The magnetizing action upon an iron rod of a *nonuniform* magnetic field, the magnetic field due to a bunched winding for example, depends * upon *the average value, along the rod, of the component of the magnetic field parallel to the rod.*

Let A be this average value, and let l be the length of the rod; then *the product lA is called the* **magnetomotive force** *along the rod.*

* The variation of the permeability μ of iron with the value of the flux density B, produces complications which need not be discussed here.

A rod passing through a magnetic field determines a certain line or path through the field and we speak of the *magnetomotive force along this path*, whether the rod is there or not.

Proposition.—The magnetomotive force along a path in a magnetic field is

$$\Omega = \frac{W}{m} \tag{68}$$

in which Ω (omega) is the magnetomotive force, and W is the work done by the magnetic field upon a magnet pole of strength m while the pole is carried along the path.

Proof.—The average value, along the path, of the component of the magnetic field parallel to the path is $A = \frac{\Omega}{l}$ from the above definition of magnetomotive force, l being the length of the path. Therefore $Am\left(= \frac{\Omega}{l} \cdot m\right)$ is the average force which pulls the pole along the path, according to equation (2), and the product of this average force by the length of the path gives the work done on the pole. That is $W = \Omega m$ or $\Omega = \frac{W}{m}$.

Q. E. D.

124. Magnetomotive force of a coil.—The most important case of the magnetization of an iron core by a bunched winding is the

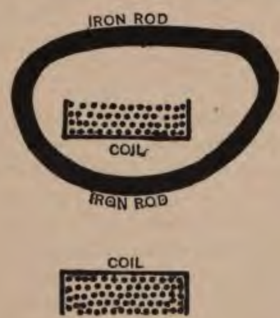


Fig. 71.

case in which the core is in the form of an endless rod which passes through, or links with, the coil as shown in Fig. 71.

The magnetomotive force along the rod in this case is entirely independent of the shape and length of the rod and of shape and size of the coil. *It depends only upon the number of turns of wire, Z, in the coil and upon the strength of the current, i, in the wire.* In fact this magnetomotive force is

$$\Omega = 4\pi Zi \tag{69}$$

where Ω is the magnetomotive force along an endless rod which links with Z turns of wire each carrying current i. When i is expressed in amperes we have

$$\Omega = \frac{4\pi}{10} Zi \tag{70}$$

This magnetomotive force along an *endless rod* which links with a coil is called the magnetomotive force of the coil.

Proof of equation (69). *Preliminary proposition.*—The amount of work, W, done in keeping the current i in a coil * constant while an additional amount of magnetic flux Φ, is set up through the opening of the coil, is:

$$W = iZ\Phi \tag{a}$$

in which Z is the number of turns of wire in the coil.

Proof of equation (*a*).—During the establishment of the flux Φ an opposing electromotive force equal to $Z\frac{d\Phi}{dt}$ is induced in the coil. In forcing the current i against this opposing electromotive force work is done at a rate equal to the product of the current into the opposing electromotive force. That is:

$$\frac{dW}{dt} = iZ\frac{d\Phi}{dt}$$

Since i is constant, this equation may be integrated, giving equation (*a*).

We shall now proceed to the proof of equation (69). Let ZZ, Fig. 72, represent a coil of Z turns of wire. Imagine NS to be a *flexible* magnet. Let the north pole of this flexible magnet be carried through the coil and around to its initial position along the dotted path. The flexible magnet will then link with the coil of wire as shown in Fig. 73. Let Ω be the magnetomotive force along the dotted path, and let m be the strength of the pole which has been carried around the path. Then according to Article 123, Ωm is the work done on the pole by the magnetic field of the coil. This work done on the moving pole by the magnetic field of the coil *is the work which is spent in keeping the current constant in spite of the electromotive force induced in the coil by the moving pole.*

* The coil may be thought of as having zero resistance for simplicity of statement.

Now the two poles of the flexible magnet are in the same positions before and after the movement, therefore, of the total number of lines of force which *radiate* from these poles, the same number pass through the coil before and after the movement. On the other hand the flux $4\pi m$ (see Art. 26), which passes along the magnet from

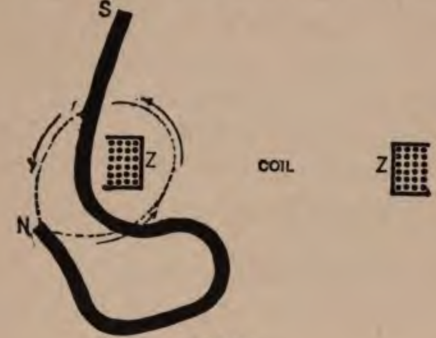


Fig. 72.

pole to pole, passes through the coil after the movement, so that the flux through the opening of the coil is increased by the amount $4\pi m$ by the movement of the pole. Therefore $iZ \times 4\pi m$ is the work spent in keeping the current constant during the

Fig. 73.

movement of the pole and, since this is equal to the work Ωm done upon the pole by the magnetic field of the coil, we have

$$\Omega m = 4\pi i Z m$$

or

$$\Omega = 4\pi Z i \qquad \text{Q. E. D.}$$

Example.—Fig. 74 shows the essential parts of a dynamo. The field magnet together with the iron of the armature constitute, practically, an *endless iron rod* which links through two coils of wire called the field coils. The combined magnetomotive force of these two coils is $4\pi Zi$ where Z is the total number of turns of wire in the two coils and i is the current in each wire.

The ampere-turn.—Equation (70) shows that the magnetomotive force of a coil is proportional to the product of amperes times turns of wire. The *ampere turn* is the magnetomotive force along a path which links with one turn of wire carrying one ampere of current. The ampere-turn is frequently used as a unit of magnetomotive force. It is necessary, however, in magnetic calculations, to reduce ampere-turns to c.g.s. units of magnetomotive force by multiplying by the factor $\frac{4\pi}{10}$ as per equation (70).

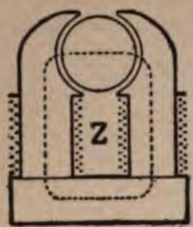


Fig. 74.

125. Intensity of magnetization.*—Let m be the strength of the magnetic pole at the end of a magnetized iron rod of sectional area q. The ratio $\frac{m}{q}$ is called the *intensity of magnetization*, $\boldsymbol{I}$, of the rod. That is:

$$\boldsymbol{I} = \frac{m}{q} \tag{71}$$

Remark.—It was shown in Article 26 that the magnetic flux emanating from a magnet pole of strength m is:

$$\Phi = 4\pi m \tag{72}$$

This flux is inwards towards a south pole and outwards from a north pole, and it is conceived to pass through the iron of the magnet from south pole to north pole. There is thus a magnetic flux $4\pi m (= 4\pi \boldsymbol{I} q)$ passing through an iron rod on account of its magnetized condition. The total flux through the rod is ordinarily greater than this, as is shown in the next article.

126. The case of a rod magnetized in a uniform field.—Consider an iron rod of sectional area q placed in a long coil of wire through which a current is sent. The magnetic field surrounding the iron rod is the resultant of two distinct parts, a and b, as follows:

* Intensity of magnetization is defined in Article 21. The definition is, however, repeated here for the sake of completeness. The student should re-read Articles 17 to 26 as an introduction to the present chapter.

a. The uniform field, $\boldsymbol{H}$, due to the coil. The lines of force of this part of the field are shown by the straight arrows in Fig. 75. On account of this part of the magnetic field $\boldsymbol{H}q$ lines of force come up to one end of the rod and pass out from the other end.

b. The magnetic field due to the magnetic poles of the rod. On account of this part of the field $4\pi m$ lines of force come up

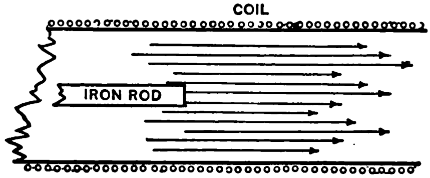


Fig. 75.

to one pole of the rod and pass out from the other pole, according to equation (72).

Therefore $\boldsymbol{H}q + 4\pi m$ lines of force come up to one end of the rod and pass out from the other end, and the total magnetic flux to be thought of as passing through the rod is

$$\Phi = 4\pi m + \boldsymbol{H}q \tag{73}$$

or

$$\Phi = 4\pi \boldsymbol{I}q + \boldsymbol{H}q \tag{74}$$

Remark.—The above discussion applies to the magnetization of a long rod in *any* uniform magnetic field. The long coil is introduced into the discussion for the sake of concreteness.

127. Flux density in iron.—The ratio $\frac{\Phi}{q}$ of the total magnetic flux through an iron rod to the sectional area of the rod is called the *flux density* or *magnetic induction*, $\boldsymbol{B}$, in the rod. That is

$$\boldsymbol{B} = \frac{\Phi}{q} \tag{75}$$

From equations (74) and (75) we have

$$\boldsymbol{B} = 4\pi \boldsymbol{I} + \boldsymbol{H} \tag{76}$$

128. Proposition.—*The flux density in an iron rod is equal to the intensity of the magnetic field in a thin slit cut across the rod.*

Proof.—A slit, AB, Fig. 76, so narrow that it does not disturb the trend of the magnetic lines, is cut across a magnetized rod. In this case the total flux $\boldsymbol{B}q$ through the iron rod crosses the

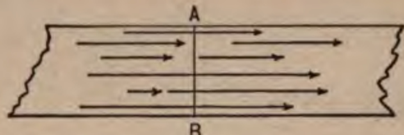


Fig. 76.

slit. Now, the flux across the slit is fq [according to equation (9)] where f is the field intensity in the slit. Therefore $\boldsymbol{B}q = fq$ or $\boldsymbol{B} = f$. Q. E. D.

129. Magnetizing field in iron.—The magnetizing field, H, at a point in an iron rod, or any mass of iron, is defined as the *resultant field intensity* which would be produced at the point by all the existing magnet poles and electric currents in the neighborhood.

Remark.—When an iron rod is placed in a magnetic field it becomes magnetized and *the magnetic field due to the poles of the rod always opposes and weakens the magnetizing field.* When the rod is very long and thin this weakening of the magnetizing field by the poles of the rod becomes negligible. When the rod forms a closed ring it has no poles and therefore no demagnetizing action on itself.

Example.—An iron rod 20 centimeters long and 3 square centimeters section, placed in a uniform magnetic field of 60 c.g.s. units intensity, is magnetized, say, to an intensity of 900 units pole per unit section *and the net value of the magnetizing field at the middle of the rod is only about 6 c.g.s. units.* Each pole of the rod is 2700 units strength ($= m$), and the two poles together produce at the middle of the rod a magnetic field of which the intensity, according to equation (3), is about 54 units, and this field is opposed to the original field so that the net intensity of the resultant field is the difference between 60 and 54.

130. Magnetization curves.—When an iron rod is subjected to a magnetizing field of greater and greater intensity, the intensity of magnetization, I, of the rod increases more and more and approaches a definite limiting value. This limiting, or saturation value, of the intensity of magnetization is different for different kinds of iron. The curves in Fig. 77 show, graphically, the re-

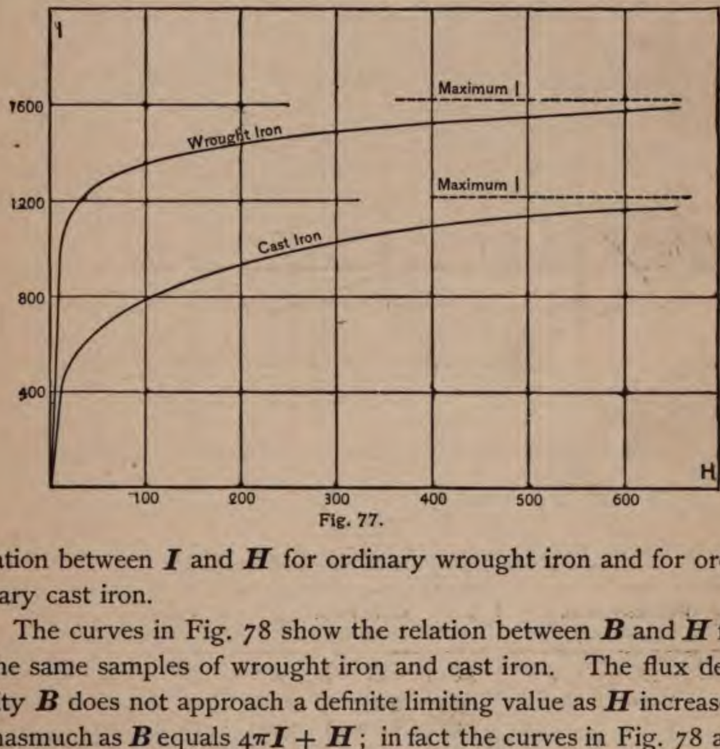


Fig. 77.

lation between I and H for ordinary wrought iron and for ordinary cast iron.

The curves in Fig. 78 show the relation between B and H for the same samples of wrought iron and cast iron. The flux density B does not approach a definite limiting value as H increases, inasmuch as B equals $4\pi I + H$; in fact the curves in Fig. 78 approach the dotted lines shown in the figure. The equation of these dotted lines is $B = 4\pi I_{max.} + H$.

131. Permeability.—Let B be the flux density in an iron rod which has been magnetized from a neutral condition by a magnetizing field H. The ratio $\frac{B}{H}$ is called the *permeability* of the iron. That is,

$$B = \mu H \tag{77}$$

in which μ is the permeability. For all kinds of iron, μ is large in value for comparatively low degrees of magnetization, and it approaches the value unity * when B and H become very great.

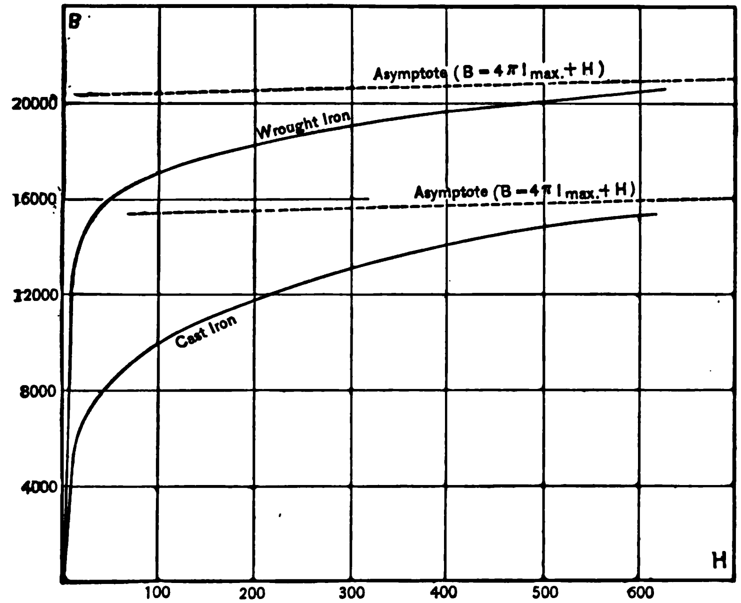


Fig. 78.

TABLE.

Magnetic properties of iron and steel.

Wrought Iron. (Hopkinson.)			Cast Iron. (M. E. Thompson.)†			Cast Steel (average). (M. E. Thompson.)		
H	B	μ	H	B	μ	H	B	μ
10	12400	1240.0	10	5000	500.0	10	9800	980.0
20	14330	716.5	20	6600	330.0	20	12450	622.5
30	15100	503.3	30	7400	246.6	30	13500	450.0
40	15550	388.8	40	7800	195.0	40	14200	355.0
50	15950	319.0	50	8450	169.0	50	14700	294.0
60	16280	271.3	60	8800	146.6	60	15100	251.6
70	16500	235.6	70	9200	131.4	70	15330	219.0

* The largest value of B hitherto obtained is about 40,000 lines per square centimeter, which was produced in wrought iron by a net magnetizing field of about 20,000 units.

† See M. E. Thompson, P. H. Knight and G. W. Bacon, *Transactions* of American Ins. of Electrical Engineers, Vol. IX. (1892).

The accompanying table gives values of $\boldsymbol{B}$, μ, and $\boldsymbol{H}$ for three kinds of iron. This table is the result of magnetic tests (see Article 136).

132. The magnetic circuit.—When an iron rod is subjected to a uniform magnetizing field along its entire length, the value of the flux density, $\boldsymbol{B}$, in the rod may be taken from the table when $\boldsymbol{H}$ is given. When, however, the iron rod varies in size at different parts of its length, or when different parts of the rod are made of different kinds of iron, or when the magnetizing field is nonuniform along the rod, then some kind of an averaging process must be used in the calculation of the flux through the rod.

In practice the iron rod usually forms nearly a complete circuit consisting in part of wrought iron, in part of cast iron and in part of an air gap; and it is desired to find the magnetomotive force required to force a prescribed amount of magnetic flux through the circuit. This calculation is carried out as follows:

Divide the prescribed magnetic flux by the sectional area of each part of the circuit, wrought iron, cast iron, or air, as the case may be. This gives the flux density $\boldsymbol{B}$ in each part of the circuit.*

Knowing $\boldsymbol{B}$ for each part of the circuit, take from the table the (average) value of $\boldsymbol{H}$ for each part. Multiply $\boldsymbol{H}$ for each part of the circuit by the length of that part. This gives the magnetomotive force required in each part of the circuit.

The sum of these magnetomotive forces gives the total magnetomotive force required. In this calculation it is to be remembered that flux density in air *is* $\boldsymbol{H}$, so that the flux density in the air gap is multiplied by the length of the air gap to give the magnetomotive force required for the air gap.

133. Work required to magnetize iron.—When an iron rod is magnetized by sending an electric current through a coil surrounding the rod an opposing electromotive force is induced in

* This assumes that all the magnetic flux passes through each portion of the iron circuit. In fact, more or less of the flux, called leakage flux, strays through the surrounding air.

the coil by the growing magnetism of the rod and *the work done in forcing the current against this opposing electromotive force is the work which magnetizes the rod.*

Proposition.—*The work, W, done in magnetizing V cubic centimeters of iron is:*

$$W = \frac{V}{4\pi}\int \boldsymbol{H} \cdot d\boldsymbol{B} \qquad (78)$$

Proof.—Consider a long* rod of iron l centimeters in length and q square centimeters in sectional area, placed in a long coil having z turns of wire per centimeter of length or lz total turns.

Let $\frac{d\Phi}{dt}$ be the rate at which the flux through the rod is increasing at a given instant. Then $lz\frac{d\Phi}{dt}$ is the opposing electromotive force induced in the coil, which, multiplied by the current i in the coil gives the rate, $\frac{dW}{dt}$, at which work is being done in magnetizing the rod. That is:

$$\frac{dW}{dt} = zli\frac{d\Phi}{dt}$$

or

$$dW = zli \cdot d\Phi \qquad (a)$$

Now, $\Phi = \boldsymbol{B}q$ or $d\Phi = q \cdot d\boldsymbol{B}$ from equation (75), and $\boldsymbol{H} = 4\pi zi$, or $zi = \frac{\boldsymbol{H}}{4\pi}$ from equation (66), so that equation (a) becomes:

$$dW = \frac{lq}{4\pi}\boldsymbol{H} \cdot d\boldsymbol{B}$$

or, since lq is equal to the volume V of the rod, we have

$$dW = \frac{V}{4\pi}\boldsymbol{H} \cdot d\boldsymbol{B} \qquad (79)$$

or

* The case of a long rod is taken in order that the demagnetizing action of the poles need not be considered.

$$W = \frac{V}{4\pi}\int \boldsymbol{H}\cdot d\boldsymbol{B}$$

Q. E. D.

Remark.—In magnetizing a short rod of iron, more work is required than is given by equation (78). This additional work goes to establish the magnetic field in the neighborhood of the free magnetic poles of the rod. Equation (78) expresses the work which is spent *within* the iron.

134. Graphical representation of work.—Let the curve opp', Fig. 79, be drawn so that the coördinates of each point of the curve represent, to scale, corresponding values of $\boldsymbol{B}$ and $\boldsymbol{H}$ for a given sample of iron when the magnetizing field at first increases from zero to a value represented by ap and then drops to zero again.

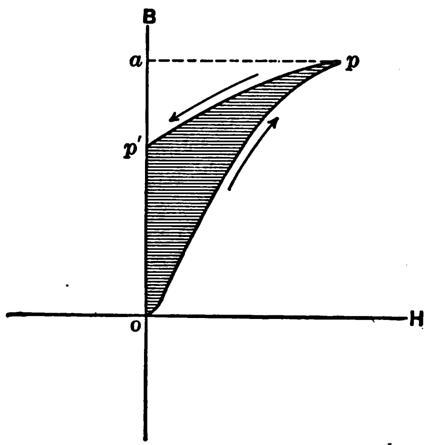


Fig. 79.

The abscissas, x, in Fig. 79, represent the magnetizing field so that

$$\boldsymbol{H} = ax$$

The ordinates, y, represent the flux density so that

$$\boldsymbol{B} = by$$

or

$$d\boldsymbol{B} = b\cdot dy$$

Substituting these values of $\boldsymbol{H}$ and $d\boldsymbol{B}$ in equation (78) we have

$$W = \frac{abV}{4\pi}\int x\cdot dy \tag{80}$$

in which a is the number of units of $\boldsymbol{H}$ represented by unit abscissa, and b is the number of units of $\boldsymbol{B}$ represented by unit ordinate in Fig. 79. Now, $\int x \cdot dy$ is the area between the curve and the Y axis. Therefore the work done in magnetizing the iron is represented by the area between the $\boldsymbol{B}$ and $\boldsymbol{H}$ curve and the Y axis.

135. Hysteresis.—The curve which represents the values of $\boldsymbol{B}$ and $\boldsymbol{H}$ during the magnetization of iron does not coincide with the curve which represents the values of $\boldsymbol{B}$ and $\boldsymbol{H}$ during demagnetization. The phenomenon indicated by this divergence of the curves of magnetization and demagnetization is called *hysteresis*. On account of hysteresis the work done in magnetizing iron is greater than the work which is regained upon demagnetization. Thus, when the given sample of iron is magnetized from o to p (Fig. 79) the work done is represented by the area oap. When the iron is then demagnetized from p to p' the work regained is represented by the area $p'ap$, so that the work lost is represented by the shaded area. A portion of this lost work is converted into heat in the iron and a portion still exists in the iron as magnetic energy due to the residual magnetism; it is difficult to say how much of the lost work has been converted into heat and how much still exists as magnetic energy.

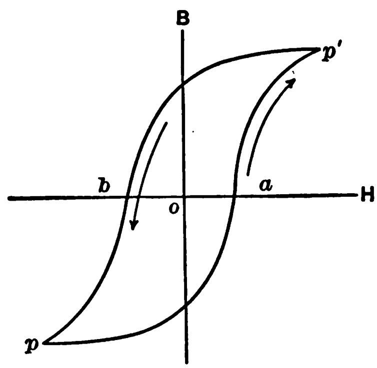


Fig. 80.

When, however, the iron is carried through a cycle* of magnetization and demagnetization, then all the work lost is changed to heat.

Thus if an iron rod be repeatedly magnetized, demagnetized, and remagnetized in the opposite sense to the same value of $\boldsymbol{B}$, a curve such as $pap'b$ (Fig. 80) will be obtained for $\boldsymbol{B}$ and $\boldsymbol{H}$. The branch pap' is for increasing values of $\boldsymbol{B}$, and the branch $p'bp$ is for decreasing values of $\boldsymbol{B}$.

* A *cycle* is a process in which a system, starting from a given state, returns to precisely the same state again at the end of the process.

The total value of $\int x \cdot dy$, equation (80), for such a magnetic cycle is the area enclosed in the loop $pap'b$, so that this enclosed area represents the loss of energy due to hysteresis during the cycle. When iron is repeatedly magnetized and demagnetized between *any* limiting values of $\boldsymbol{B}$, a curve very similar to Fig. 80 is obtained.

Steinmetz has found that the energy, W, lost in iron per cycle can be expressed with sufficient accuracy for practical purposes by the formula:

$$W = nV\boldsymbol{B}^{1.6} \tag{81}$$

in which V is the volume of the iron in cubic centimeters, $\pm\boldsymbol{B}$ are the limits of the cycle, and n is a constant depending upon the quality of the iron. For annealed refined wrought iron $n =$ .002. This formula gives W in ergs.

The following table gives the hysteresis loss W in ergs per cubic centimeter per cycle for various ranges of $\boldsymbol{B}$ for annealed refined wrought iron.

TABLE.

Hysteresis loss per cycle in annealed wrought iron.

$W\left(\frac{\text{ergs}}{\text{c.c.}}\right)$ (After Swinburne.)	$\pm\boldsymbol{B}$
650	2500
1600	5000
3200	7500
5000	10000
7200	12500
9600	15000

Hammering, or stretching iron beyond its elastic limits, greatly increases the hysteresis loss for a given range of $\boldsymbol{B}$, and in cast iron and hardened steel the hysteresis loss is vastly greater than in annealed wrought iron. Hysteresis losses affect very materially the efficiency of transformers, the iron cores of which are carried through from 40 to 150 magnetic cycles per second. For this reason the best annealed refined wrought iron is used for transformer cores and for the armatures of dynamos.

136. Ewing's method of testing iron.—A long slim rod, AB, Fig. 81, of the iron to be tested is placed in a vertical position in a long vertical coil of wire. The upper end, A, of the rod is on a level with, and at a distance d due east of a small suspended magnet at M. Let l be the length and q the sectional area of the iron rod. Let z be the number of turns of wire per centimeter length of the long coil, and let H' be the intensity of the horizontal component of the earth's magnetic field at M.

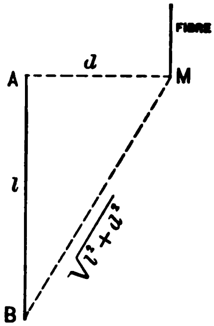


Fig. 81.

A measured current, i, is sent through the magnetizing coil and the intensity of the magnetizing field is

$$\boldsymbol{H} = 4\pi zi \qquad (a)$$

from equation (66).

It remains to find the flux density $\boldsymbol{B}$ produced in the rod by this magnetizing field. Let m be the strength of the pole A and $-m$ the strength of the pole B of the magnetized rod. These poles deflect the suspended magnet M through the angle ϕ and the flux density in the rod is determined from this observed deflection as follows:

The horizontal field at M due to the pole A is $\frac{m}{d^2}$, and the horizontal component of the field at M due to the pole B is $-\frac{m}{l^2+d^2} \times \frac{d}{\sqrt{l^2+d^2}}$, according to equation (3). Therefore the total horizontal field at M due to the magnetized rod is

$$h = \frac{m}{d^2} - \frac{md}{(l^2+d^2)^{\frac{3}{2}}} \qquad (b)$$

This field h is at right angles to the earth's horizontal field H', as shown in Fig. 82, and the suspended magnet turns through the angle ϕ and points in the direction of the resultant field. From Fig. 82 we have

$$h = H' \tan \phi \qquad (c)$$

Substituting this value of h in equation (b) we have

$$H' \tan \phi = \frac{m}{d^2} - \frac{md}{(l^2 + d^2)^{\frac{3}{2}}} \qquad (d)$$

This equation gives m when ϕ, d and l are observed and H' known. From the value of m thus obtained, $\boldsymbol{I}$ becomes known from equation (71), and $\boldsymbol{B}$ becomes known from equation (76).

Fig. 82.

A complete test of a sample of iron consists of the determination of a whole series of values of $\boldsymbol{B}$ and $\boldsymbol{H}$ in this way.

The most troublesome errors in this method are as follows:

1. The magnetizing field $\boldsymbol{H}$ is somewhat less than $4\pi zi$ because of the demagnetizing action of the rod upon itself. This error is small when the rod is very long in comparison with its diameter. This error may be accurately allowed for if the test piece is in the form of an ellipsoid.

2. The current in the magnetizing coil acts directly upon the suspended magnet and produces some deflection, whereas equation (d) assumes that the deflection is due entirely to the field which emanates from the magnet poles of the rod. This error is easily allowed for.

3. The poles A and B are not concentrated at the ends of the rod but spread over considerable portions of the ends of the rod. This source of error is in part provided against by placing the end A of the rod slightly higher in level than the suspended magnet M, and by taking l in equation (d) rather less than the actual length of the rod.

137. The ballistic method of testing iron (Rowland).—A ring of the iron to be tested (A, Fig. 83), of section area q and peripheral length l, is wound uniformly with Z turns of wire through which the magnetizing current i is sent. Another coil of n turns, not shown in the figure, is wound upon the ring and connected with a ballistic galvanometer. The magnetizing current, furnished by a battery B, flows through an ammeter C, a rheostat R, so arranged as to enable the observer to produce quick changes in the current, and a reversing switch S.

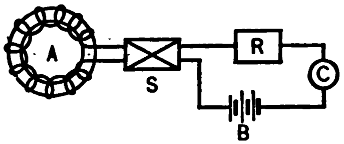


Fig. 83.

The magnetomotive force around the iron ring is $4\pi Zi$, according to equation (69), which, divided by the peripheral length of the ring, gives the average magnetizing field $\boldsymbol{H}$. That is

$$\boldsymbol{H} = \frac{4\pi Zi}{l} \qquad (a)$$

If the current i is suddenly changed by a measured amount Δi, the change in $\boldsymbol{H}$ is

$$\Delta\boldsymbol{H} = \frac{4\pi Z}{l} \cdot \Delta i \qquad (b)$$

The corresponding change in the flux density $\boldsymbol{B}$ is

$$\Delta\boldsymbol{B} = \frac{kra}{nq} \qquad (c)$$

in which k is the reduction factor of the ballistic galvanometer, r is the total resistance of the ballistic galvanometer circuit, and a is the observed throw of the ballistic galvanometer needle produced by the change in $\boldsymbol{B}$.

Proof of equation (c).—The changing flux through the iron ring induces, in the n turns of wire an electromotive force equal to $n \cdot \frac{d\Phi}{dt}$. This electromotive force produces, in the ballistic galvanometer circuit, a current which by Ohm's law is equal to $\frac{\text{electromotive force}}{\text{resistance}}$, and which is equal to the rate, $\frac{dQ}{dt}$, at which charge passes through the ballistic galvanometer. Therefore

$$\frac{dQ}{dt} = \frac{n}{r} \cdot \frac{d\Phi}{dt}$$

or

$$Q = \frac{n}{r} \cdot \Delta\Phi \qquad (d)$$

where Q is the charge that passes through the ballistic galvanometer while the magnetic flux through the iron changes by the amount $\Delta\Phi$ or while the flux density $\boldsymbol{B}$ changes by the amount

$$\Delta \boldsymbol{B} = \frac{\Delta\Phi}{q} \qquad (e)$$

From the equation to the ballistic galvanometer [equation (87)] as explained in Chapter X. we have

$$Q = ka \qquad (f)$$

Substituting the value of $\Delta\Phi$ from equation (e), and the value of Q from equation (f) in equation (d), and solving for $\Delta \boldsymbol{B}$, we have

$$\Delta \boldsymbol{B} = \frac{kra}{nq} \qquad \text{Q. E. D.}$$

The curve of $\boldsymbol{B}$ and $\boldsymbol{H}$ is plotted from a series of observed values of $\Delta \boldsymbol{H}$ and $\Delta \boldsymbol{B}$ as follows: Beginning at any point, we lay off $\Delta \boldsymbol{H}$ and $\Delta \boldsymbol{B}$ to scale. This determines the next point of the curve. From this point we lay off the next observed values of $\Delta \boldsymbol{H}$ and $\Delta \boldsymbol{B}$, thus locating the next point of the curve, and so on. The whole curve is thus drawn and the axes of $\boldsymbol{B}$ and $\boldsymbol{H}$, if it is desired to indicate them, can be drawn through the center of the figure parallel to $\Delta \boldsymbol{B}$ and $\Delta \boldsymbol{H}$ respectively.

The most troublesome errors in this method are the following:

1. The magnetizing field, equation (a), is greater in the inner portion of the ring where l is smaller. This lack of uniformity in the magnetizing field introduces complications not considered in equations (b) and (c), and these equations will, therefore, in general, give erroneous results. These errors are in great part obviated by using a ring of such dimensions that l differs but little in various parts of it, and by using a mean value for l in equation (a).

2. Equation (c) takes account only of changes in $\boldsymbol{B}$ which occur promptly, during an interval of time which is but a fraction of the time of vibration of the ballistic needle.

In some kinds of iron (very soft wrought iron) a quick change in $\boldsymbol{H}$ produces a prompt change in $\boldsymbol{B}$, followed by a sluggish change which continues for a few seconds. Equation (c) in this case leads to slightly erroneous results.

Any slow change in the magnetizing current, due for example to heating of the wires in circuit or to the polarization of the battery, produces a corresponding slow change in $\boldsymbol{H}$ and $\boldsymbol{B}$, in which case also the use of equation (c) leads to erroneous results.

CHAPTER IX.

THERMOELECTRIC CURRENTS.

138. Seebeck's discovery.—In 1821 Seebeck found that an electric current is produced in a circuit of two metals when one of the junctions of the two metals is warmer than the other junction. Thus, if the ends of a short piece of iron wire are soldered to copper wires leading to a galvanometer, the galvanometer gives a deflection when one of the soldered junctions is heated. Such an arrangement is called a *thermoelement.*

The thermopile.—The electromotive force of a single thermoelement seldom exceeds a few thousandths of a volt even when the two junctions are at widely different temperatures. A number of thermoelements may, however, be connected in series. Such an arrangement is called a *thermopile.* Fig. 84 shows the essential features of a thermopile. *AAAA* are bars of one metal and *BBBB* are bars of another metal. Junctions 1, 3, 5 and 7 are heated while junctions 2, 4 and 6 are kept cool, or *vice versa.* Melloni, in his classic experiments on radiant heat, used a thermopile built up with bars of antimony and of bismuth.

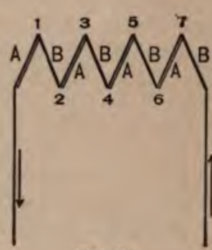


Fig. 84.

139. The Peltier effect.—In 1834 Peltier discovered that heat* is generated or absorbed at a junction of two metals when a current flows across the junction ; generated when the current flows in one direction, and absorbed (that is, the junction becomes

* Aside from the heat generated in accordance with Joule's law, Art. 41.

cool) when the direction of the current is reversed. For strong currents this *Peltier effect* is masked by the heat generated on account of electrical resistance, for the rate of generation of heat by the Peltier effect is proportional to the current, while the rate of generation of heat on account of resistance is proportional to the square of the current. The Peltier effect is most easily shown as follows: A current from a voltaic battery is sent through a thermopile. This current heats one set of junctions and cools the other set, and the thermopile will give a reversed current if it is disconnected from the battery and connected to a galvanometer.

140. The Thomson effect.—Lord Kelvin, from the principles of thermodynamics, was led to suspect the existence of a cooling action (or heating action, according to the direction of the current) when an electric current flows along a wire of which the temperature is not uniform. This he found to be the case. In some metals, copper, for example, the electric current causes an absorption of heat (a cooling effect) at a given point when the current flows in the direction in which the temperature is increasing, and *vice versa*. In iron, on the other hand, absorption of heat takes place when the current flows from hot to cold.

141. Thermoelectric power.—Consider a thermoelement of given metals with both junctions at a given temperature T. Let Δe be the electromotive force of the element when the temperature of one junction becomes $T + \Delta T$. Then, for small values of ΔT, the electromotive force Δe is proportional to ΔT. That is

$$\Delta e = U \cdot \Delta T \tag{82}$$

in which U is the proportionality factor. This quantity U is called the *thermoelectric power* of the given pair of metals at the given temperature.

Example.—A thermoelement of iron and copper with both junctions at 0° C. has an electromotive force of about 0.0000021 volt when one junction is $\frac{1}{10}$° C. warmer or cooler than the other. When one junction is $\frac{2}{10}$° C. warmer than the other the electromotive force of the element is about 0.0000042 volt, so that the thermoelectric power of copper and iron at 0° C. is 0.000021 volt per degree difference of temperature of the two junctions. At 100° C. the thermoelectric power of copper and iron is about 0.0000085 volt per degree.

Let e be the electromotive force of a thermoelement of which the junctions are at the temperatures T' and T'' respectively, then:

$$e = \int_{T'}^{T''} U \cdot dT \tag{83}$$

This equation is not a direct result of equation (82), but it depends, in addition, upon the fact that the electromotive force of a thermoelement, whose junctions are at

T and T'', is the sum of the electromotive forces of the element when its junctions are at T and T', and at T' and T'' respectively, T' being a temperature between T and T''.

142. Thermoelectric diagram.—The ordinates between the various curves (straight lines) in Fig. 85 represent the thermoelectric powers of the respective pairs

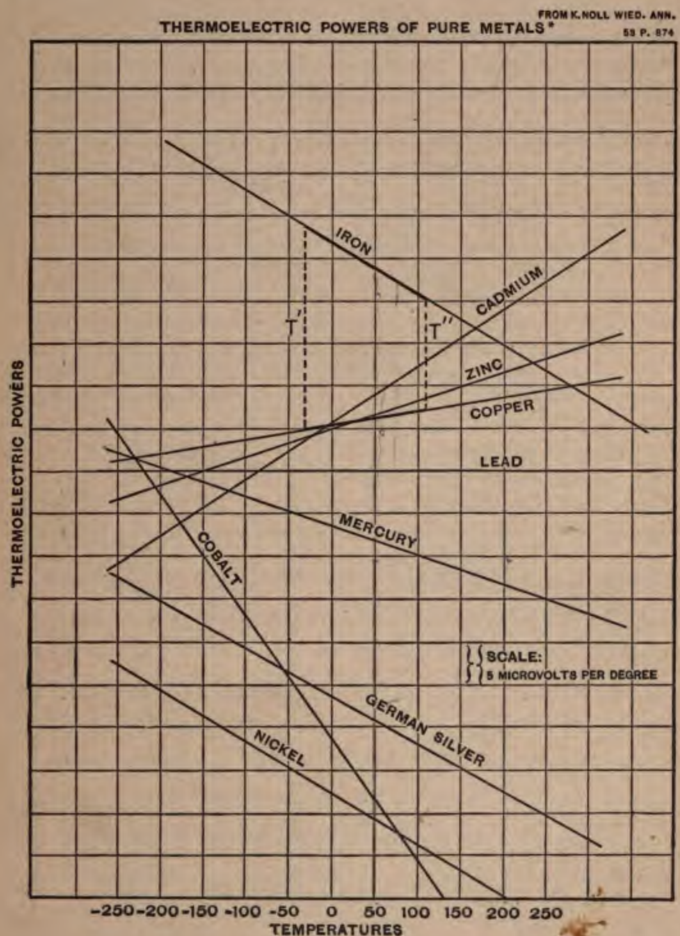


Fig. 85.

of metals at various temperatures. These lines are called the thermoelectric lines of the respective metals, and the figure is called the thermoelectric diagram. These thermoelectric lines are, of course, determined by observation of the thermoelectric power of the respective pairs of metals at various temperatures, and they are approximately straight. The possibility of representing the thermoelectric powers of various pairs of metals in one diagram grows out of the fact that if U' is the thermoelectric power, say, of iron-lead, and U'' the thermoelectric power of lead-nickel at the same temperature, then $U' + U''$ is the thermoelectric power of iron-nickel at that temperature.

It follows from equation (83) that the electromotive force of a thermoelement, say of iron and copper, with its junctions at T' and T'' respectively, is represented by the *area* enclosed between the thermoelectric lines for iron and for copper, and the ordinates whose abscissas represent the temperatures T' and T''. Areas on opposite sides of the point of intersection of two thermoelectric lines are considered as opposite in sign.

Example.—The thermoelectric power of copper and iron at $-50°$ C. is 25.5 microvolts per degree. At 150° C. the thermoelectric power is 10.0 microvolts per degree. The mean thermoelectric power between -50 C. and 150° C. is $\frac{25.5 + 10}{2}$ or 17.7 microvolts per degree, which, multiplied by the temperature difference, 200°, gives 3540 microvolts for the electromotive force of a copper-iron element with one junction at $-50°$ C. and the other junction at 150° C.

143. Neutral temperature for two metals.—The temperature at which the thermoelectric power of two metals is zero is called the *neutral temperature* for those two metals; it is the temperature corresponding to the point of intersection of the thermoelectric lines of the two metals. Thus the neutral temperature of cadmium-iron is about 170° C., the neutral temperature of copper-zinc is about 30° C. When the temperature of one junction of a thermoelement is as much above the neutral temperature of the element as the temperature of the other junction is below neutral temperature, then the electromotive force of the element is zero.*

144. The parabolic formula for thermoelectromotive force.—With one junction of a thermoelement kept at a fixed temperature, and the other junction at temperature T, the electromotive force of the element may be quite accurately represented by the formula:

$$e = a + bT + cT^2 \tag{84}$$

in which a, b and c are constants depending upon the metals employed, and upon the constant temperature of the one junction. This is called the parabolic formula inasmuch as it is the equation to a parabolic curve.

Discussion of equation (84).—Let n be the neutral temperature of the element and t the fixed temperature of the one junction. The thermoelectric lines being sen-

* In this statement it is assumed that the thermoelectric lines are straight, which is not exactly true.

sibly straight, the enclosed area between the temperatures n and t is proportional to $(n-t)^2$ or equal, say, to $k(n-t)^2$, and the enclosed area between temperatures n and T is, similarly, $k(n-T)^2$. The total electromotive force is represented by the difference of these two areas so that $e=k(n-t)^2-k(n-T)^2$. Expanding this expression we find a constant term, a term involving T, and a term involving T^2.

145. Use of the thermoelement for the measurement of temperature.—The constants a, b and c of equation (84) being determined* for a given temperature, say 0° C. of the one junction of the thermoelement, the temperature T of the other junction may be calculated when e is observed. The thermoelement furnishes in this way a very convenient means for the determination of temperature. Equation (84) is not rigorously exact, and therefore a thermoelement which is to be used for very exact temperature measurements must be calibrated. For this purpose the electromotive force of the element is observed for a series of accurately known temperatures and a curve showing the relation between electromotive force and temperature is drawn.

For the measurement of very high temperatures a thermoelement of pure platinum wire, and a wire of platinum-rhodium or platinum-iridium alloy is generally used.

146. The thermopile as an electric generator.—Many attempts have been made to use the thermopile commercially as a generator of electric current, but these attempts have failed, mainly because of the extremely low efficiency; not more than a fraction of one per cent. of the heat employed to warm the one set of junctions being converted into electrical energy.

In the thermoelectric battery of Clamond, the elements are made of strips of German silver soldered to blocks of an alloy of antimony and zinc. The electromotive force of a single element of this kind when one junction is at ordinary room temperature and the other junction is heated nearly to the melting point of antimony-zinc alloy, is about 0.04 volt. Clamond arranged a

*These constants are determined by observing three pairs of values of e and T, which values, substituted in equation (84), give three simultaneous equations from which a, b and c may be determined.

number of such elements in rings one above the other so as to form a hollow cylinder. The inner face of this cylinder, which is

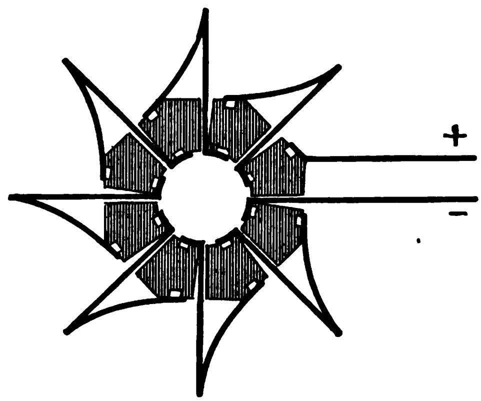


Fig. 86.

made up of the junctions to be heated, is subjected to the action of a gas flame. A single ring of elements is shown in Fig. 86.

CHAPTER X.

ELECTRIC CHARGE. THE CONDENSER.

147. Electric charge.—According to the hydraulic analogue employed in Article 27, an electric current in a wire may be looked upon as a transfer of electricity along the wire very much as a current of water in a pipe is a transfer of water along the pipe.

Let Δq be the amount of water, which, during the time interval Δt, flows past a point in a pipe. The quotient $\frac{\Delta q}{\Delta t}$ is the *rate of flow* of water in the pipe, and this rate of flow may be spoken of as the strength, i, of the water current. If the water current is constant in strength, then the quantity, q, of water which flows past a point in t seconds is:

$$q = it$$

Similarly, the quantity, q, of electricity which flows past a point on a wire carrying a constant current may be defined as the product of strength, i, of the electric current and the time. That is:

$$q = it \tag{85}$$

If the electric current is not constant in strength it is necessary to consider the small quantity, Δq, of electricity which flows past a given point during the short interval of time Δt. In this case $\Delta q = i \cdot \Delta t$ or

$$\frac{\Delta q}{\Delta t} = i \tag{86}$$

That is, the electric current in a wire may be defined as the rate of flow of electricity past a given point on the wire.

Quantity of electricity is usually spoken of as *electric charge* or simply as *charge.*

Remark.—In hydraulics *quantity of water* is the fundamental and easily measured quantity, and *water current* is most conveniently defined as *quantity of water per second.* In case of electricity the fundamental and easily measured quantity is *electric current,* and *quantity of electricity* is most conveniently defined as the product of *electric current multiplied by time.*

This hydraulic analogue is, of course, incomplete, as pointed out in Article 27.

Units of electric charge.—The quantity of electricity transferred in one second by one ampere is called a *coulomb;* q in equation (85) is expressed in coulombs when i is expressed in amperes and t in seconds. When, in equation (85), i is expressed in c.g.s. units and t in seconds, then q is expressed in c.g.s. units of charge. One c.g.s. unit of charge is equal to ten coulombs.

Another unit of charge is defined in Article 179.

148. The measurement of charge. The ballistic galvanometer. —A very large electric charge may be determined by observing the time, t, during which the given charge will maintain a sensibly constant measured current. The charges most frequently encountered in practice are, however, too small to be measured in this way; for the measurement of such charges the ballistic galvanometer is used, as follows:

The charge to be measured is sent through a galvanometer as a very short pulse of electric current which sets the needle of the galvanometer swinging. The maximum deflection, a, of the needle at the first swing is called the *throw* of the galvanometer and *this throw, if not too large, is proportional to the amount of charge q which is carried through the galvanometer by the pulse of current.* That is:

$$q = ka \tag{87}$$

The quantity k is called the *reduction factor* of the galvanometer. This reduction factor is usually determined in practice by observing the throw a due to a known charge.

Equation (87) is strictly true only for a galvanometer with a heavy needle, or with a heavy moving coil in case of the D'Arsonval type of instrument, which is not subject to any perceptible air friction as it vibrates. Such a galvanometer is called a *ballistic galvanometer.*

Theory of the ballistic galvanometer.—The following discussion applies to a galvanometer of which the suspended magnet is small compared with the size of the coils.

The coils of the galvanometer are mounted with their plane vertical and in the direction of a horizontal magnetic field of intensity H; for example, the earth's field or a field produced by governing magnets. A current i in the coils produces a magnetic field at the center of the coils which is at right angles to H, and of which the intensity is

$$f = Gi \tag{i}$$

where G is the constant of the coils. Let M be the magnetic moment of the galvanometer magnet. The field f, perpendicular to the axis of the needle, exerts upon it a torque equal to fM or to GiM, equation (5); and this torque, being unbalanced, is equal to the product of the moment of inertia, K, of the magnet into its angular acceleration. Therefore, writing $\frac{dq}{dt}$ for i, we have

$$K\frac{d\omega}{dt} = GM\frac{dq}{dt} \tag{ii}$$

in which ω is the angular velocity, and $\frac{d\omega}{dt}$ the angular acceleration of the suspended magnet. If ω is zero at the instant of closing the galvanometer circuit, equation (ii) gives*

$$K\omega = GMq \tag{iii}$$

which is the fundamental equation of the ballistic galvanometer. If G, M, and K were known, q could be calculated from an observed value of ω.

In order to render the ballistic method feasible it is necessary to avoid the determination of the moment of inertia, K, and the determination of the magnetic moment,

* This simple case of integration, plainly stated, is as follows: Equation (ii), if we divide both terms by GM, gives $\frac{K}{GM}\frac{d\omega}{dt} = \frac{dq}{dt}$. It means in this form that *q increases $\frac{K}{GM}$ times as fast as ω.* Therefore, *if ω and q start from zero together, q must always be $\frac{K}{GM}$ times as large as q*, which is equation (iii).

M, of the needle, and also to avoid the impracticable operation of observing the angular velocity, ω, of the needle. This is accomplished as follows:

A. From equation (7) we have

$$\frac{4\pi^2 K}{\tau^2} = MH \qquad \text{(iv)}$$

B. A constant known current I sent through the ballistic galvanometer will produce a permanent deflection ϕ of the needle, such that

$$\tan\phi = \frac{GI}{H} \qquad \text{(v)}$$

for the current I produces a field of intensity GI at right angles to H, and the needle turns into the direction of the resultant field.

C. The kinetic energy of the moving needle after it has attained angular velocity ω is $\frac{1}{2}K\omega^2$. If there is no damping of its motion, the needle will turn until all this energy is spent in pulling it out of the direction of the field H. To move the north pole N (Fig. 87), the strength of which is m, from a_1 to b, and the south pole S of like strength, over a similar path, requires work against the force Hm. This work is

$$2Hm \times a_1a_2 = HM(1 - \cos\theta)$$

since $a_1a_2 = \frac{l}{2}(1 - \cos\theta)$ and $M = ml$, l being the length of the needle. Therefore we have

$$\tfrac{1}{2}K\omega^2 = HM(1 - \cos\theta) \qquad \text{(vi)}$$

in which θ is the angle through which the needle is thrown by the discharge q.

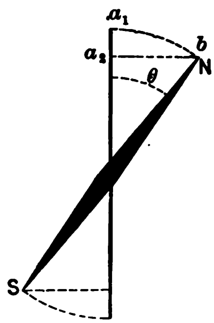


Fig. 87.

Using equations (iv), (v), and (vi), the quantities G, $\frac{K}{M}$, and ω may be eliminated from (iii), giving, when solved for q,

$$q = \frac{2I\tau}{\pi \tan\phi} \sin\tfrac{1}{2}\theta \qquad (88)$$

This equation expresses q in terms of easily observed quantities; *viz.*, the permanent angle of deflection, ϕ, produced by a known current, I, the time of vibration, τ, of the needle, and the angle, θ, through which the needle is thrown by the passage of the charge q through the galvanometer.

If the value of q is to be free from errors other than those involved in the determination of I, τ, ϕ, and θ, the conditions implied in the equations (iii), (iv), (v), and (vi) must be complied with. A galvanometer constructed with a view to the realization of these conditions is properly a ballistic galvanometer. These conditions are as follows:

Conditions implied by equation (iii).

1. The needle must be at rest when the discharge through the coils begins.
2. The directing field, H, must be in the plane of the coils.
3. The field, f, due to the current in the coils must remain sensibly perpendicular

to the axis of the needle during the entire discharge, so that the torque may be equal to GIM, or $GM\frac{dq}{dt}$, throughout the whole time of its action. This condition requires a slow vibration of the needle (τ large), except in cases when the duration of the discharge is very brief.

Conditions implied by equation (iv).

1. The suspending fiber must be free from torsion.

2. The damping must be so slight as to have no sensible influence upon the time of vibration of the needle.

Conditions implied by equation (v).

1. The diameter of the coils must be large compared with the length of the needle; in other words, the law of the tangent galvanometer must hold true for such deflections as are produced by the steady current I.

Conditions implied by equation (vi).—The whole of the energy ($\frac{1}{2}K\omega^2$) must be employed in turning the needle against the directing field (H). This means that there must be no damping, a condition which cannot be realized in practice. For the failure to fulfil this condition corrections must be applied.

Working formula.—For work not requiring extreme accuracy equation (88) may be greatly simplified. Let a be the observed throw, in scale divisions, produced by the discharge. Let b be the permanent deflection, in scale divisions, produced by the current I, and let D be the distance, also in scale division, of the scale from the mirror. Then we have, approximately, $\sin\frac{1}{2}\theta = \frac{a}{4D}$, and $\tan\phi = \frac{b}{2D}$, whence equation (88) becomes

$$q = \frac{I\tau}{\pi b} \cdot a$$

or

$$q = ka \qquad (87)\ bis$$

149. Electrokinetics and electrostatics.—The electric current is electricity in motion and the study of the phenomena of the electric current is called *electrokinetics*. The study of the phenomena of electricity at rest is called *electrostatics*.

150. Electrically charged bodies. *Preliminary statements.*—Consider two bodies of metal, A and B, Fig. 88, which are connected, as shown, to the terminals of a battery, or to any source of electromotive force. When the wire is connected a momentary pulse of electric current flows through the wire out of one body into the other and the bodies A and B are said to become *charged with electricity*. The quantity of charge, q, which passes through the wire may be measured by allowing the current pulse to flow through a ballistic galvanometer. The body into which the charge flows is said to become *positively charged* and the body out of which the charge flows is said to become *negatively charged*, that is the charge on the one body is $+q$ and on the other body is $-q$. Electrically charged bodies always occur thus in pairs, the positive charge on one body being always associated with an equal negative charge on some other body or bodies.

The dielectric. The electric field.—The region between the two charged bodies A and B, Fig. 88, is, of course, filled with some electrical insulator such as air, oil, or glass. An insulator between two charged bodies is called a *dielectric*. This dielectric is the seat of a peculiar stress called *electric field*, similar, in many respects, to magnetic field. The lines of force of this electric field trend somewhat as shown in the figure, touching the surfaces of A and B at right angles. These lines of force are thought of as *going out from* the positively charged body, and *coming in towards* the negatively charged body.

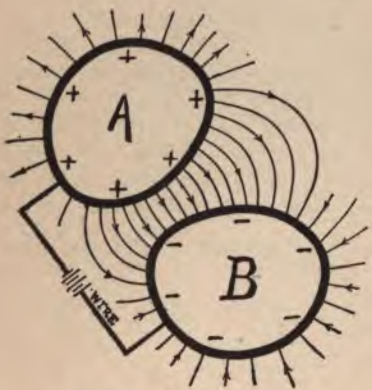


Fig. 88.

Electrostatic attraction.—The charged bodies A and B, Fig. 88, attract each other. This attraction, called *electrostatic attraction*, shows that the lines of force of an electric field are in a state of tension and have a tendency to shorten. This tension of the lines of force pulls outwards on the surface of A and of B at each point. This outward pull on the surface of a charged body is very strikingly shown by pouring a viscid liquid over the sharp lip of a charged ladle. The liquid is pulled into fine jets by the lines of force which emanate from the liquid as it passes over the lip. When melted rosin is used in this way the jets congeal into very fine fibers which float about in the air.

Need of very large electromotive forces.—The phenomena described above, and in fact most of the phenomena of electrostatics, are easily perceptible only when the bodies are charged by electromotive forces of many thousands of volts. The most convenient means for producing these large electromotive forces is the Holtz or the Wimshurst electrical machine. In all diagrams, however, a battery will be shown where it is desired to indicate an electromotive force.

The need of good insulation.—The large electromotive forces used in experiments with charged bodies make it necessary to insulate the bodies well, especially if the bodies are to retain their charges for any length of time after the battery, or other source of electromotive force, is disconnected. This insulation is usually accomplished by mounting the bodies upon pillars of glass or of ebonite. Glass has a tendency to condense atmospheric moisture upon its surface, which destroys its insulating power. This difficulty is partly overcome by covering the surface of the glass with varnish. When every care is taken to insulate charged bodies the charges are slowly dissipated when the bodies are left to themselves.

Two charged bodies, for example, A and B, Fig. 88, lose their charges almost instantaneously when they are connected by a wire, and the electric field disappears at the same time. A line of force never begins and ends on the same conductor, nor upon two conductors which are brought into contact or connected by a wire.

151. Electrostatic capacity. The condenser.—The amount of charge, q, which flows out of B and into A, Fig. 88, when the battery is connected is larger the greater the electromotive force of the battery. In fact, this charge is proportional to the electromotive force of the battery, as may be readily shown by means of a ballistic galvanometer. Therefore we may write:

$$q = CE \qquad (89)$$

where q is the charge that is drawn out of B and forced into A, Fig. 88, by a battery of which the electromotive force is E, and C is a constant depending upon the size and shape of A and B and upon the nature of the intervening dielectric. This quantity C is called the *electrostatic capacity* of the pair of bodies A and B.

If the bodies A and B are in the form of metal plates separated by a thin layer of dielectric their electrostatic capacity is large. Such an arrangement is called a *condenser*. Condensers of great capacity are made by sheets of tinfoil separated by sheets of waxed paper or mica. Such condensers are used quite extensively in ocean and land telegraphy and in telephony.

The *Leyden jar* is a condenser made by coating the inside and outside of a glass jar with tinfoil.

Units of capacity.—A condenser is said to have a capacity of one *farad* when one coulomb of charge is drawn out of one plate and forced into the other plate by an electromotive force of one volt; C in equation (89) is expressed in farads when q is expressed in coulombs and E in volts. The *farad* is an enormously large capacity compared with capacities ordinarily met with in practice, therefore the *microfarad* (one millionth of a farad) is a more convenient unit.

When, in equation (89), q is expressed in c.g.s. units of charge and E in c.g.s. units of electromotive force, then C is expressed in c.g.s. units of capacity. The c.g.s. unit of capacity is equal to 10^9 farads.

Electric absorption.—When a condenser, which has been charged for some time, is discharged and then left standing, additional charge collects on the condenser plates so that a second or third discharge can be taken from the condenser. It seems as though a portion of the initial charge on the condenser were *absorbed* by the dielectric, this absorbed charge being slowly given back to the condenser plates when these have been discharged.

152. The mechanical analogue of the condenser.—A and B, Fig. 89, represent two cavities in an extended elastic solid such, for example, as rubber. If these cavities are filled with water and connected to a pump by means of a pipe, the pump will draw a certain amount of water out of one cavity and force it into the other cavity, causing one cavity to contract and the other to expand, and causing the surrounding mass of rubber to be strained, the lines of stress being somewhat as shown in the figure. The greater the difference of pressure generated by the pump the greater the quantity of water, q, which is drawn from one cavity and forced into the other; in fact this quantity of water is proportional to the pressure difference generated by the pump.

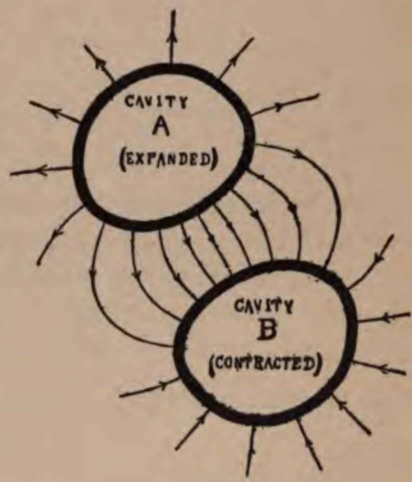


Fig. 89.

If the two cavities A and B are *near together and separated by a thin layer only of the rubber*, then a small pressure difference generated by the pump will draw a large quantity of water out of B and force it into A, causing a great distortion of the thin layer of rubber. This thin layer of rubber between the two cavities is analogous to the thin layer of dielectric between the metal plates of a condenser.

153. Inductivity * of dielectrics.—The capacity of a condenser, with plates of given size and at a given distance apart, depends upon the dielectric. The quotient: *capacity of a condenser with given dielectric divided by the capacity of the same condenser with air between its plates* is called the *inductivity* of the dielectric. For example, the inductivity of petroleum is about 2.04, that is, the capacity of a given condenser is about 2.04 times as great when the dielectric is petroleum as it is when the dielectric is air. A condenser is called an *air condenser*, a *mica condenser*, a *paraffine condenser*, *etc.*, according to the dielectric used between the plates.

* Sometimes called specific inductive capacity.

The accompanying table gives the inductivities of a few dielectrics.

TABLE.

Inductivities of various substances.

Glass	3 to 10	Shellac	2.95 to 3.60
Sulphur	2.24 to 3.84	Mica	4 to 8
Vulcanite	2.50	Quartz	4.5
Paraffin	1.68 to 2.30	Turpentine	2.15 to 2.43
Rosin	1.77	Petroleum	2.04 to 2.42
Wax	1.86	Water	73 to 90

154. Dependence of the capacity of a condenser upon the size and distance of its plates.—Using a ballistic galvanometer as described in Article 151, it may be shown experimentally that the capacity, C, of an air condenser is proportional to the area, a, of its plates, and inversely proportional to the distance, x, between its plates. That is, C is proportional to $\frac{a}{x}$, so that we may write

$$C = b \cdot \frac{a}{x} \tag{90}$$

in which C is the capacity of the air condenser, a is the area of one plate, x is the distance between the plates, and b is a constant. When C is expressed in c.g.s. units of capacity, a in square centimeters, and x in centimeters, then the value of b, as determined by experiment, is $\frac{1}{4\pi v^2}$, where $v = 3 \times 10^{10} \frac{\text{cm}}{\text{sec}}$.*

If the condenser plates are separated by a dielectric of which the inductivity is k, then the capacity is k times as great and equation (90) becomes

$$C = kb \cdot \frac{a}{x} \tag{91}$$

If the capacity of the condenser is to be expressed in farads equation (91), using the numerical value of b, becomes

$$C_{\text{farads}} = 885 \times 10^{-16} \times \frac{ka}{x} \tag{92}$$

* This is precisely the velocity of light in air. The significance of this fact is explained in Article 250.

in which k is the inductivity of the dielectric, x is the thickness of the dielectric in centimeters, and a is the area in square centimeters of one plate of the condenser or the area of the thin sheet of dielectric.

155. Measurement of capacity.—The capacity of a condenser may be determined directly by measuring, with a ballistic galvanometer, the charge q that is transferred from one plate of the condenser to the other plate by a battery of known electromotive force E. The ratio $\frac{q}{E}$ then gives the value of the capacity, according to equation (89).

A variety of indirect methods have been devised for measuring capacity. See Gray's *Absolute Measurements*, Vol. I., p. 418.

The capacity of a condenser may be calculated from its dimensions by equation (92) where the specific inductive capacity of the dielectric is known.

156. Work done by an electromotive force during the transference of a given charge.—Consider an electromotive force, E, producing a current, i, in a circuit. The rate at which this electromotive force does work is Ei, which, multiplied by a time t, gives the work done during that time, so that $W = Eit$. But the product it is the charge transferred during the time t. Therefore

$$W = Eq \qquad (93)$$

in which W is the work done by an electromotive force E in transferring the charge q.

157. The energy of a charged condenser.—A charged condenser stores energy in very much the same manner that a bent spring stores energy. The energy of a charged condenser is usually much less than the work done in charging the condenser, for, when an electromotive force, E, is connected to an uncharged condenser, charge rushes into the condenser and surges back and forth, causing a large portion of the work Eq to be dissipated. In order that all the work done by the electromotive force may be utilized in charging the condenser without any dissipation, the electromotive force must have a value zero when it is connected, and must be made to increase slowly in value. The work done in charging a condenser in this way is equal to the potential energy stored in the charged condenser.

The potential energy of a charged condenser is:

$$W = \frac{1}{2}\frac{q^2}{C} \tag{94}$$

or

$$W = \frac{1}{2}Eq \tag{95}$$

or

$$W = \frac{1}{2}CE^2 \tag{96}$$

Proof.—Let $q\,(= Ce)$ be the charge on the condenser plates when the growing electromotive force has reached the value e, and let Δq be the additional charge, which is introduced into the condenser by a slight increase of the charging electromotive force. Then $\Delta W = e \cdot \Delta q$ is the work done in producing this increase of charge, but $e = \frac{q}{C}$ from equation (89), so that

$$\Delta W = \frac{1}{C} q \cdot \Delta q$$

Integrating this expression from $q = 0$ to $q = q$ we have

$$W = \frac{1}{2}\frac{q^2}{C}$$

Equations (95) and (96) are obtained by writing CE for q, and C^2E^2 for q^2, respectively, in equation (94).

CHAPTER XI.

THE ELECTRIC FIELD AND ITS USE IN THE EXPLANATION OF ELECTROSTATIC PHENOMENA.

158. Intensity of electric field. *Transference of charge by a moving ball.*—A and B, Fig. 90, are two metal plates connected to a battery of electromotive force E, and b is a light metal ball suspended between A and B by a silk thread. When once this ball is started it continues to vibrate back and forth from plate to plate, and at each movement the ball carries across a definite amount of charge $\pm q$. This charge carried across by the ball is replaced by the battery. In doing this the battery does an amount of work Eq, and *this work reappears as mechanical work done on the ball as it is pushed across by the electric attraction or repulsion.* Let F be the force which pushes on the ball, then Fx is the work done by this force in pushing the ball across from plate to plate,* so that $Fx = Eq$ or

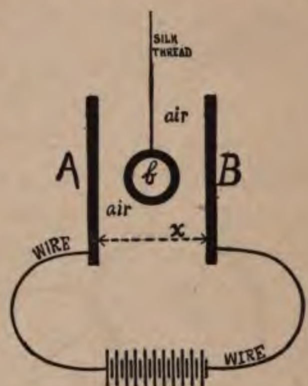


Fig. 90.

$$F = \frac{E}{x} \cdot q \qquad (97)$$

Any region in which a charged body is acted upon by a force† *is called an electric field;* thus the region between the plates A and

* The ball is supposed to be quite small.

† That is, a force which depends upon the charge and does not exist when the body has no charge.

B, Fig. 90, is an electric field inasmuch as the charged ball, b, is pulled to the right or left according as the charge on the ball is positive or negative.

The force, F, with which an electric field pulls on a charged body (small) placed at a given point in the field is proportional to the charge, q, on the body, so that

$$F = fq \tag{98}$$

in which f is the proportionality factor. This quantity f is called the *intensity of the electric field* at the given point.

Comparing equations (97) and (98) it is evident that the intensity of the electric field between the parallel plates AB, Fig. 90, is:

$$f = \frac{E}{x} \tag{99}$$

That is, electric field is expressed in terms of *electromotive force per unit length* (in *volts per centimeter*, for example).

Direction of field.—The direction in which the force in an electric field pulls on a positively charged body is adopted as the *direction of the field.* A *line of force* in an electric field is a line drawn so as to be in the direction of the field at each point.

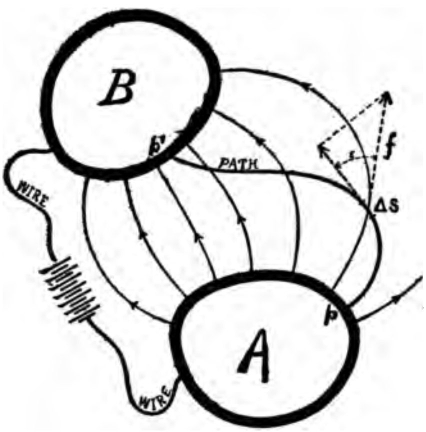


Fig. 91.

159. General relation between electromotive force and electric field.—Consider two charged conductors A and B, Fig. 91, connected as shown to a battery of electromotive force E. Suppose that an amount, q, of charge is transferred from A to B along any path, pp'. The work done by the battery in replenishing the charge is

$$W = Eq \qquad (93)\ bis$$

and *this is equal to the work done by the force with which the electric field pushes on the charge while the charge is passing along the path.*

Consider an element Δs of the path. Let f be the intensity of the electric field at Δs, and let e be the angle between f and Δs, as shown in the figure. Then qf is the force pulling on the charge q as it passes along the element Δs, $qf \cos e$ is the resolved part of this force parallel to Δs, and $qf \cos e \cdot \Delta s$ is the work done by this force as the charge passes along the element. Therefore, the total work done by the electric field upon the charge q as it is carried from p to p' is

$$W = \Sigma qf \cos e \cdot \Delta s$$

or

$$W = q\Sigma f \cos e \cdot \Delta s$$

Comparing this expression with equation (93) above, it is evident that the electromotive force along a given path in an electric field is

$$E = \Sigma f \cos e \cdot \Delta s \qquad (100)$$

That is, each element of the path is multiplied by the resolved part parallel to the element of the electric field intensity at the element, and the sum of these products is the electromotive force along the path. The equation $f = \frac{E}{x}$ (99) or $E = fx$, as applied to the uniform electric field between parallel plates, is a special case of equation (100).

160. Electric potential.—The electromotive force * between two points in an electric field is called the *difference of potential* or the *difference of electric pressure* between those points.

Potential at a point.—Any point or region, the earth, for example, may be chosen arbitrarily as the region of zero potential. Then the *potential at a point* may be defined as the electromotive force * between that point and the arbitrarily chosen point

* When the electromotive force between two points is not the same for different paths connecting the points, then one cannot speak of electromotive force as a difference of potential. This matter is fully discussed in Chapter XVII.

or region of zero potential. Equation (100), when applied to a path from the region of zero potential to a given point, gives the potential at the point.

Equipotential surfaces.—The electromotive force is zero along a path which is at each point at right angles to an electric field, inasmuch as cos e in equation (100) is zero at each point of the path. Therefore the potential has the same value at all points of such a path. Likewise the potential has the same value at

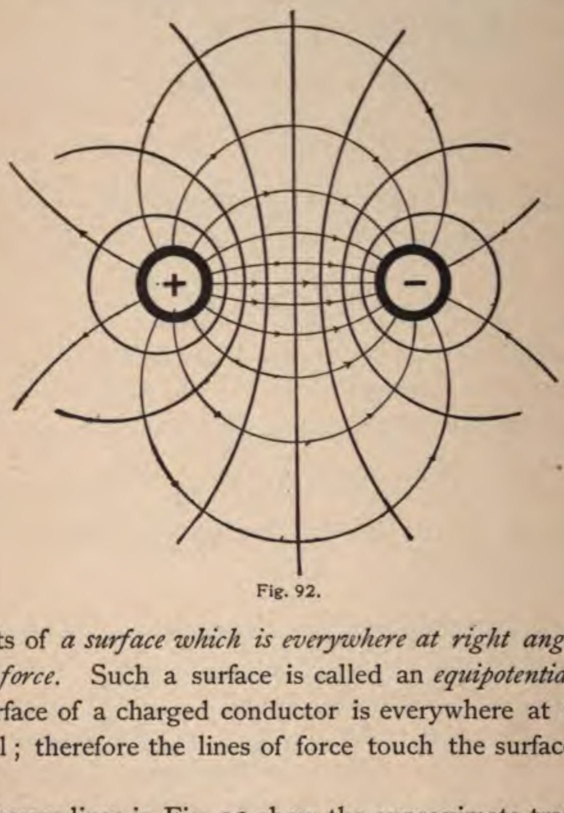


Fig. 92.

all points of *a surface which is everywhere at right angles to the lines of force.* Such a surface is called an *equipotential surface.* The surface of a charged conductor is everywhere at the same potential; therefore the lines of force touch the surface at right angles.

The heavy lines in Fig. 92 show the approximate trend of the surfaces of equipotential in the region surrounding two oppositely charged spheres. The lines of force are marked by arrow heads. In the neighborhood of an isolated charged sphere the equipotential surfaces are spherical surfaces concentric with the charged sphere. In the region between charged parallel plates the equipotential surfaces are planes parallel to the plates.

161. Electroscopes.—An electroscope is a device for indicating the existence of an electric charge on a body, or for detecting an electric field.

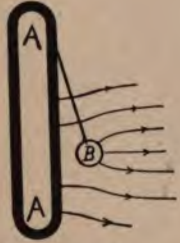


Fig. 93.

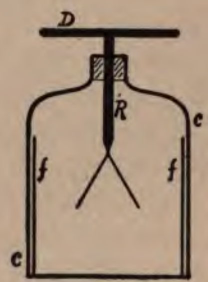


Fig. 94.

The pith ball electroscope consists of a gilded ball of pith suspended by a silk thread.

This ball, when charged, is pulled in the direction of the lines of force in an electric field. This affords an easy means for detecting an electric field and for tracing the lines of force.

A pith ball may be hung alongside of a body *AA*, as shown in Fig. 93. The pith ball then takes a portion of any charge on the body and the lines of force which emanate from the ball deflect it, as shown in the figure.

The gold-leaf electroscope consists of a metal disk, *D*, and rod, *R*, Fig. 94, from the lower end of which two gold leaves are hung side by side. The whole is supported in a glass case, *cc*, which protects the gold leaves from air currents. The sides of *cc* are lined with strips of metal foil, *ff*, to increase the sensitiveness. When the disk, rod and gold leaves are charged the leaves are pulled

Fig. 95.

apart by the lines of force as shown in Fig. 95, which for clearness shows the instrument without a glass case.

The action on the gold-leaf electroscope of a charged body brought near to the plate *D* is briefly as follows:

1. *When the electroscope has no initial charge* some of the lines of force pass from the body *into* the disk and then *out from* the leaves to the strips, *ff*, causing the leaves to diverge. If the body is removed the electroscope again becomes neutral. If, while the charged body is near *D*, the disk or rod is touched with the hand the lines of force passing *out from* the leaves cease to exist and the leaves fall together. If now the charged body is removed the lines of force going into the disk from the charged body spread over disk, rod and leaves, the leaves diverge, and the electroscope is left charged. This operation, called charging by influence, is explained more fully in Article 167.

(2) *When the electroscope has an initial charge*, say a positive charge, then a positively charged body brought near to *D* pushes the initial charge down into the leaves, as it were, and the divergence of the leaves is increased. If a negatively charged body is brought near to *D* the positive charge is pulled up into the disk, as it were, and the divergence of the leaves is decreased. If the negatively charged body is brought nearer, the leaves will come together; and if the body is brought still nearer, the leaves will again diverge.

This action of a charged body upon a gold-leaf electroscope affords a convenient means for detecting and identifying positive and negative charges.

162. Electric charge resides wholly on the surface of a charged conductor. *Electrical screening.*—The electrostatic phenomena exhibited by charged conductors are precisely the same whether the bodies are solid or hollow; and, if the bodies are hollow, not the slightest effect of the charges can be detected inside of them, however thin their walls may be. *The lines of force of the electric field end, therefore, at the surface of the charged conductors* or, as it may be said, *the electric charges reside wholly on the surfaces of charged conductors.*

A conducting shell (for example a metal box) screens its interior completely so that no action of any kind reaches the interior from charged bodies outside. Thus a shell of metal *C*, Fig. 96, screens its interior completely. The lines of force which touch the shell *C* end at its surface. The *ending* on *C* of the lines of force from *A* is *negative charge*, and the *beginning* on *C* of lines of force which reach *B* is *positive charge*.

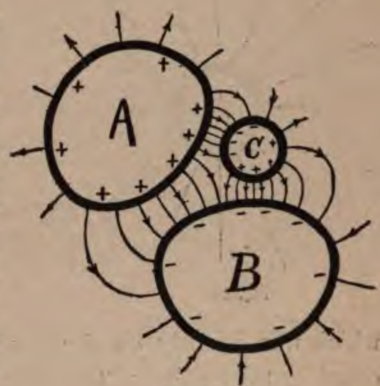


Fig. 96.

The fact that electrical field does not penetrate into a conductor shows that conductors cannot sustain the peculiar kind of stress which constitutes electrical field and therefore this electrical stress cannot be transmitted from the outside to the inside of a metal box.

163. Mechanical analogue of electrical screening.—Consider a solid body *B*, Fig. 97, entirely separated from the surrounding solid by empty space *eee*. Stress and distortion of the surrounding solid cannot affect *B* in any way, and, conversely, stress and distortion of *B* cannot affect the surrounding solid, for the empty space is incapable of transmitting the stress. This empty space in its behavior towards mechanical stresses is analogous to a conductor in its behavior towards electrical stresses (electrical field).

Fig. 97.

164. A charged conductor shares its charge with another conductor placed in contact with it.—Fig. 98 shows the lines of force in the neighborhood of a charged conductor, *A*. When another conductor, *B*, is placed in contact with *A* the lines of force arrange themselves as shown in Fig. 99. The charge

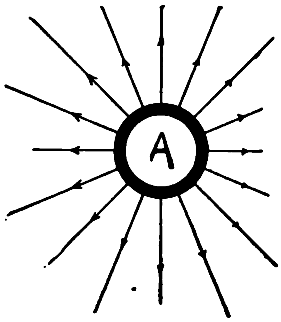


Fig. 98.

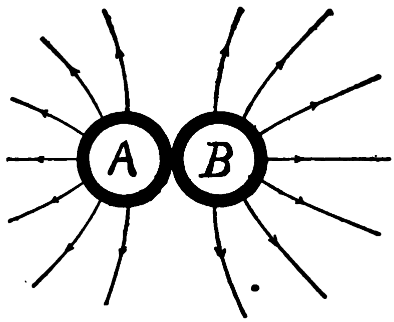


Fig. 99.

which was initially on *A* spreads over both *A* and *B*, as indicated by the *ending* of the lines of force.

165. Faraday's experiment.—A charged body, *B*, Fig. 100, is lowered into a metal vessel and the opening of the vessel is closed

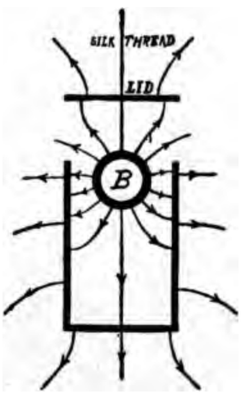


Fig. 100.

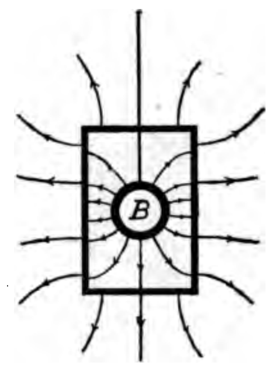


Fig. 101.

with a metal lid. As the body is lowered into the vessel each line of force emanating from *B* is cut in two, as it were, by the ll of the vessel, so that, when *B* is entirely enclosed by the vessel *as many lines of force emanate from the external surface of the vessel as from the body B, and all the lines of force which emanate from B terminate on the inner surface of the vessel.* Therefore, if $+q$ is the amount of charge on B, $-q$ is the amount of charge on the inner surface of the vessel, and $+q$ is the amount of charge on the external surface of the vessel.

Fig. 100 shows the state of the electrical field while the body B is being lowered into the vessel, and Fig. 101 shows the state of the electrical field when the body B is enclosed by the vessel and its lid.

Further, the distribution of the electrical field outside of the vessel does not depend in any way upon the position of the body

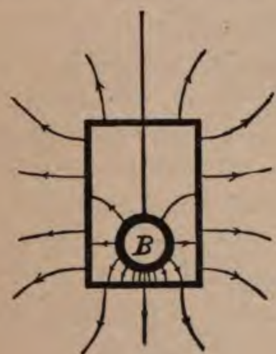


Fig. 102.

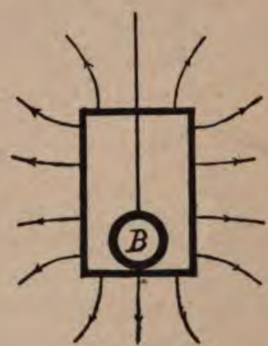


Fig. 103.

B inside of the vessel. Thus, in Figs. 101 and 102, the distribution of the electric field (lines of force) is very different inside while the external field is the same.

If the body B (a conductor) is brought into contact with the wall of the vessel, the charges on B and on the inner wall of the vessel disappear, while the external charge and external field are not affected as shown in Fig. 103.

166. Giving up of entire charge by one body to another.—When the body B, Figs. 100, 101, 102 and 103, is lowered into the vessel and allowed to touch the interior it loses all its charge and remains without charge when removed from the vessel, while the charge left on the outside of the vessel is equal to and of the same sign as the original charge on B. The body B may thus be said to give up its entire charge to the vessel.

167. Charging by influence.—Let A, Fig. 104, be a charged body, then the neighborhood of A is an electric field. Let B be a conductor brought near to A. This body B takes on positive and negative charges where the lines of force end upon it as shown. If a conducting body, C, is brought into contact with B as shown in Fig. 105, then the bodies B and C are charged as shown in the figure, *and the bodies B and C retain these charges when they are separated and removed to a distance from A.*

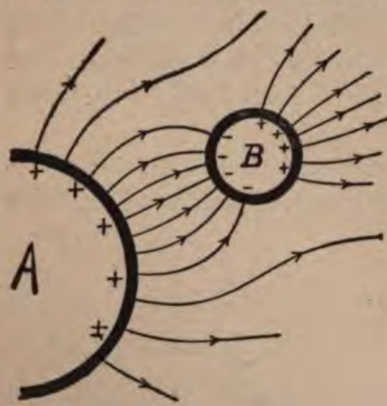


Fig. 104.

This operation is called *charging by influence*, equal amounts of positive and negative electricity being produced. It often occurs that one is interested only in the charging of the body B, in which case the hand may serve instead of the body C.

168. Charging by contact and separation. Contact electromotive force.—Many substances when separated, after having been brought into intimate contact by rubbing them together, are charged with electricity. Thus rubbing glass with silk charges the glass positively and the silk negatively; rubbing rosin with fur charges the rosin negatively and the fur positively. This phenomenon is called *charging by contact and separation;* it is explained as follows:

Any two substances left in contact settle to a state of equilibrium * with a definite electromotive force between them. This

* That is to say, a state in which there is no tendency to further change of any

electromotive force is called the *contact electromotive force* of the two substances.

Example.—Two flat plates of copper and zinc connected momentarily with a wire settle to a difference of potential of about

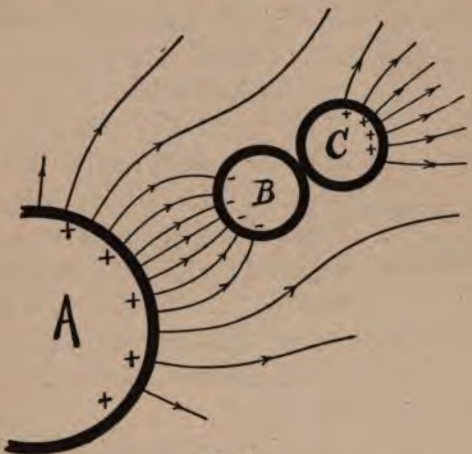


Fig. 105.

$\frac{9}{10}$ of a volt. If the plates are at a distance of .01 centimeter apart, the region between them will be a uniform electric field of intensity $\frac{9}{10}$ volt divided by .01 centimeter, or 90 volts per centimeter, according to equation (99).

If the plates are moved apart while connected with a wire, the electromotive force between the plates remains constant and the electric field intensity falls off.

If, however, the plates are insulated so that no charge can escape from either plate, then the field intensity, $\frac{E}{x}$ [see equation (99)], remains constant as the plates are separated, and therefore the electromotive force between the plates increases as x increases. Thus, if the plates are separated from a distance of .01 centimeter to a distance of 10 centimeters, the electromotive force between them increases from $\frac{9}{10}$ of a volt to 900 volts.

When metal plates are charged by contact and separation, great care must be taken to prevent the plates touching after they have been slightly separated, for this will allow the electromotive force between the plates to fall immediately to the value of the contact electromotive force. No such difficulty exists in the charging of nonconductors by contact and separation.

169. The electrophorus.—This is a device for the production of a charge by influence. It consists of a plate, *D* (Fig. 106), of rosin or hard rubber, which has been electrified (negatively) by beating it with a piece of fur or flannel, and a disk *M* of metal provided with an insulating handle *H*. When the metal disk is brought near to the negatively charged plate of rosin and touched with the finger, it is left with a charge of positive electricity. This charge remains on *M* as a free charge when *M* is removed to a distance from *D*. This operation may be repeated indefinitely.

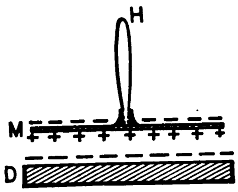


Fig. 106.

With the electrophorus only small quantities of electric charge can be produced. It is possible, however, to devise apparatus for the *continuous generation* of charge. Such a device is called an **electrical machine.** There are two types: (*a*) frictional machines, in which the method of contact and separation is employed; (*b*) influence machines, in which the operation is essentially that of the electrophorus.

170. The frictional electric machine.—This machine, in its most approved form, consists of a rotating glass disk, *DD* (Fig. 107), the various parts of which come in succession into intimate contact with two leather cushions *AA*, smeared with an amalgam of tin, zinc, and mercury. The surface of the glass plate as it leaves these cushions is left highly charged with positive electricity, while the cushions are left negatively electrified. The negative charge flows into the insulated conductor, *N*, which is connected with the cushions by means of the springs *SS*. The positive charge carried on the surface of the glass disk, is collected by the points of the metal combs, *CC*, and flows into the insulated conductor *P*. Two silk aprons, *pp*, one on each side of the rotating disk tend to prevent the escape of the positive charge from the surface of the disk.

171. Influence electric machines.—The electrophorus is the simplest arrangement for the generation of charge by influence.

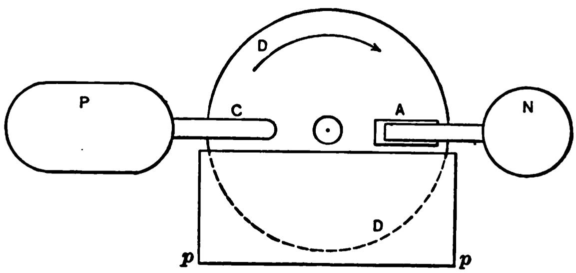


Fig. 107 a.

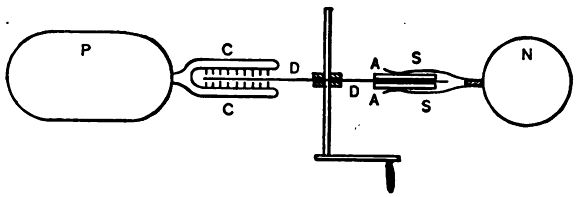


Fig. 107 b.

As has already been stated, the various influence machines are essentially similar to it in action, except that the *inducing* charge is generated by the machine also. The *revolving doubler* is the oldest form. The Holtz machine, next in order, was modified by Töpler, and the result is the *Töpler-Holtz machine*, now extensively used. The *Wimshurst machine*, which is perhaps the simplest of all, is also much used.

The reversibility of influence electric machines.—The various influence electric machines may be used as *electric generators*, as described below, in which case they must be supplied with mechanical power and they deliver electric charge at high electromotive force; or they may be used as *electric motors*, in which case they must be supplied with electric charge at high electromotive force from some outside agent and they deliver mechanical power. A large portion of the mechanical power used to drive an influence machine is lost in friction, and when such a machine is driven as a motor especial care must be taken to reduce the friction in order that the machine may run.

172. The revolving doubler.—Two plates of metal, C and D, Fig. 108, called *carriers*, are mounted on an insulating arm and

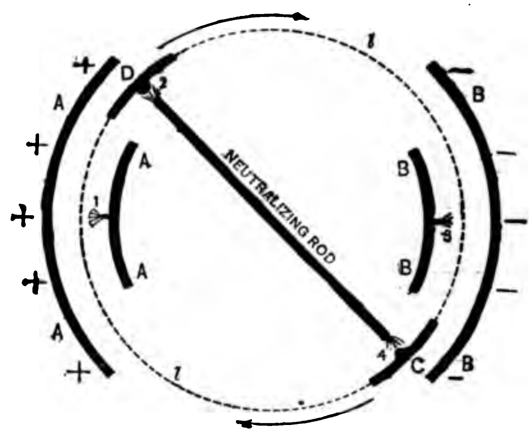


Fig. 108.

rotated so as to pass along the dotted line, *ll*. When C and D are in the positions shown in the figure they are touched momentarily by the metal brushes 2 and 4, which are fixed to the ends of a stationary metal neutralizing rod. Under the influence of the charged conductors AA and BB, the two carriers take on positive and negative charges respectively. The rotation then carries C into the interior of the hollow conductor AA, where it is momentarily touched by the metal brush 1 and gives up its entire charge to AA. The carrier D at the same time gives up its entire charge to BB in a similar manner. As the carriers pass out from AA and BB they are again momentarily touched by the brushes 2 and 4, taking on fresh charges, which they give up to A and B as before, and so on.

The infinitesimal charges imparted to C and D by the mere contact of the brushes is sufficient, because of the multiplying action of the machine, to bring the arrangement very quickly into active operation even when A and B have been allowed to lose every vestige of charge. The machine is for this reason said to be *self-exciting*.

173. The Töpler-Holtz machine.—The action of the Töpler-Holtz machine is very similar to the action of the revolving doubler. In the Töpler-Holtz machine, the two charged bodies, AA and BB, of the revolving doubler consist, each, of two *sepa-*

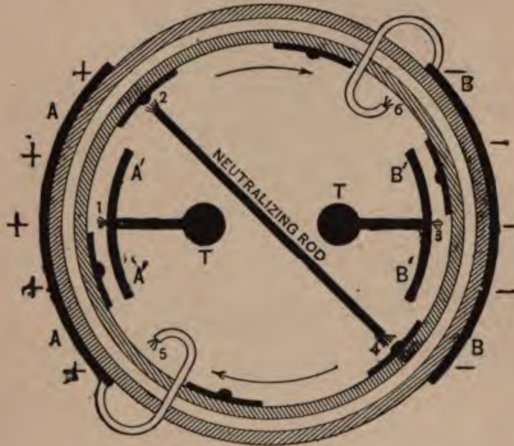


Fig. 109.

rate parts, A and A', and B and B', as shown in Fig. 109. Just before the carriers enter the regions between A and A' and between B and B' they touch the brushes 5 and 6 and *give up a portion of their charges to A and to B*, the remainder of their charges is given up to A' and to B' when they touch the brushes 1 and 3, Fig. 109.

TT are the terminals of the machine and the charge which is drawn from the machine is taken from A' and B', the charge being left on A and B so that the machine may continue in active operation.

The machine in its usual form consists of a circular disk of varnished glass upon which the six to ten or twelve carriers are fixed. These carriers are made of disks of tin foil and they are provided with metal buttons which are touched momentarily by the various metal brushes as the glass disk revolves. Behind this rotating disk is a fixed glass disk upon which are strips of foil, or paper, serving as the *inductors* A and B, Fig. 109. In front of the rotating disk the collectors A' and B' are placed.

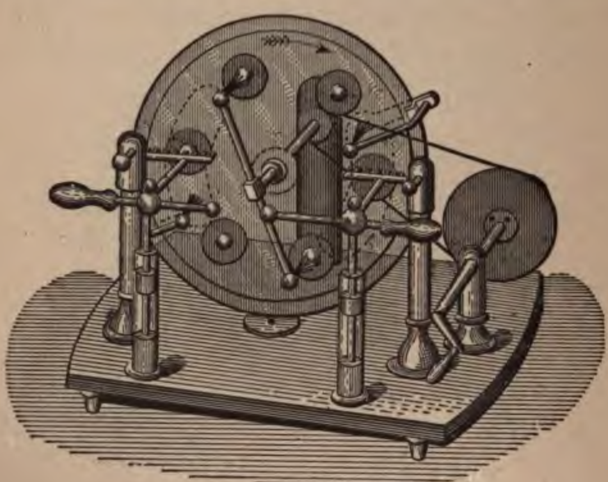


Fig. 110.

The essential features of the machine are best represented by a diagram of the kind shown in Fig. 109, where the metal carriers and the inductors A and B are arranged on glass cylinders.

The Töpler-Holtz machine is self-exciting. It is not so much affected by moisture as the frictional machine. A perspective view of the Töpler-Holtz machine is shown in Fig. 110.

174. The Wimshurst machine. *Preliminary.*—Let P, Fig. 111, be a metal point, connected to earth, near a charged surface AB. Let CD be a sheet of glass. The lines of electric stress from the harge AB converge upon the point P, being very little disturbed by the presence of the glass sheet, *CD*. The electric field in the neighborhood of *P* is thus intense enough, if *AB* is at all strongly

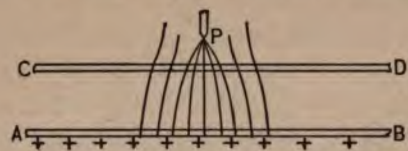


Fig. 111.

charged, to break down the dielectric, namely, the air between the point *P* and the plate of glass *CD*. After the break-down of

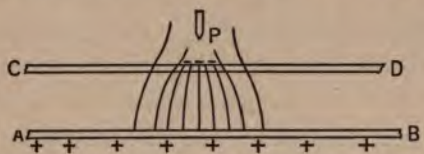


Fig. 112.

the dielectric the lines of force end on the surface of *CD* as *negative charge*, as shown in Fig. 112.

The small portion of the surface of *CD* which faces the point is thus negatively charged, and the *amount of charge on this small portion is equal to the amount of positive charge on the much larger part of AB, from which the lines emanate which have been broken down between P and CD.* If the plate *CD* is moved to the left, fresh lines of electric stress will crowd between the point and *CD*, and by their continual breaking down, the surface of *CD* as it moves out from under *P* will be left *much more strongly charged*

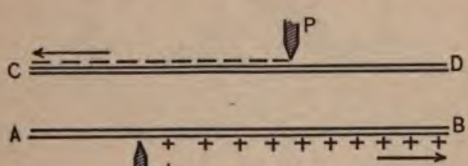


Fig. 113.

than the plate *AB*. This plate *AB* may itself be charged by moving it to the right under a point *P′* under the inducing action of CD, as shown in Fig. 113. The charges on AB and CD will thus grow more and more intense until checked by the rapidly increasing leakage from the surface of the plates. The negative charge on CD after it has passed well beyond the point P', and the positive charge on AB after it has passed well beyond the point P, may be collected by metal combs and used for any purpose.

Fig. 114.

The practical form of the Wimshurst machine consists of two varnished disks of glass side by side, driven in opposite directions, with neutralizing rods and collectors properly arranged. This arrangement is shown in Fig. 114, which for the sake of clearness shows the machine as consisting of two cylinders of glass rotated in opposite directions.

The Wimshurst machine is frequently made with metal carriers in the form of radial strips of tin foil pasted to the rotating disks,


Fig. 115.

and metal brushes are used at the ends of the neutralizing rods. When so arranged the machine is easily self-exciting.

Fig. 115 shows a perspective view of a Wimshurst machine.

CHAPTER XII.

ELECTROSTATIC ATTRACTION. ELECTROMETERS.

175. Electric flux.—The product of the intensity of an electric field into an area at right angles to the direction of the field is called the *electric flux* across the area. That is:

$$\Phi = fa \tag{101}$$

in which Φ is the electric flux across a square centimeters of area at right angles to an electric field of intensity f.

The *unit of electric flux* is the flux across one square centimeter of area at right angles to an electric field of unit intensity. This unit is called *the line of force* or simply *the line*. See Articles 25 and 26 for statements concerning magnetic flux which hold also for electric flux.

176. Amount of electric flux which emanates from an electric charge.—It was first shown by Gauss that *the amount of electric flux, or the number of lines of force, which emanates from an electric charge is strictly proportional to the charge*. The simplest case is that in which the charge is spread uniformly over a flat surface, as on the flat metal plates of an air condenser. The following is a discussion of this case.

Substitute the value of C from equation (90) in equation (89) and we have:

$$q = ba \cdot \frac{E}{x} \tag{102}$$

Now, according to equation (99), $\frac{E}{x}$ is the intensity of the electric field between the plates of the condenser, so that $a \cdot \frac{E}{x}$ is the electric flux, Φ, from plate to plate, according to equation (101); therefore equation (102) becomes:

$$q = b\Phi \qquad (103)$$

in which b is the constant mentioned in Article 154. This equation (103) is entirely general, as was first shown by Gauss; that is, *the electric flux, Φ, which emanates from charge q is always equal to $\frac{q}{b}$.* In case q is a positive charge the flux *passes out from it*, in case q is a negative charge the flux *comes into it.*

177. Transformation of equation (103). A new set of electrical units.—The numerical value of a given physical quantity varies according to size of the unit in terms of which the quantity is expressed; thus a given length may be 1.623 kilometers, 1623 meters, or 162300 centimeters. The larger the unit the smaller the number which expresses a given physical quantity. Now, equation (103) shows that a given charge sends out a perfectly definite amount of electric flux; but the numerical values of q and Φ, and therefore the numerical value of the factor b, depend upon the size of the unit of charge and of the unit of flux respectively; and by properly choosing these units the factor b * may be made to have any assigned numerical value.

In Article 26 it was shown that the amount of magnetic flux outwards from a magnet pole of strength m is equal to $4\pi m$ *and it is desirable, in dealing with electrostatic theory, to choose our units so that the relation between electric charge and electric flux may be identically the same as the relation between magnetic charge (magnetic pole) and magnetic flux.* That is, our new units are to be so chosen that equation (103) becomes

$$q = \frac{1}{4\pi}\Phi$$

or

$$\Phi = 4\pi q \qquad (104)$$

* It must be remembered that the factor b is a physical constant, like the density ·ater or the velocity of sound, and its real value is not a matter of convention.

That is, the quantity b is to have the value $\frac{1}{4\pi}$ instead of the value $\frac{1}{4\pi v^2}$ or *this quantity is to be made v^2 times as large* by the change of units.

By inspection of equation (102) this is seen to require any one of the following changes in the units of charge and electromotive force :

a. The number which expresses a given electromotive force must be made v^2 times as small, that is, the unit electromotive force must be chosen v^2 times as large as the unit heretofore used, unit charge being unchanged ; or

b. The number which expresses a given electromotive force must be made v times as small and the number which expresses a given charge must be made v times as large. That is, the unit electromotive force must be chosen v times as large and the unit charge v times as small as the units heretofore used ; or

c. The number which expresses a given charge must be made v^2 times as large, that is, the unit charge must be made v^2 times as small as the unit heretofore used, unit electromotive force being unchanged.

So far as the immediate object of reducing the value of the factor b to $\frac{1}{4\pi}$ is concerned, *a*, *b* and *c* are equally satisfactory; but another consideration is important, namely, that the work, Eq [see equation (93)], done by an electromotive force in transferring a charge may still be equal to the product Eq where electromotive force and charge are expressed in terms of the new units. The scheme *b* satisfies this condition, inasmuch as the numerical value of a charge is increased in the same ratio that the numerical value of an electromotive force is decreased. That is, the numerical value of the product Eq is not affected by this change of units. A simple and direct definition of the new unit charge will be given later. For brevity these new electrical units (which are also c.g.s. units) will be called *Faraday units* to distinguish them from the units which have been heretofore, and are to be hereafter, spoken of as c.g.s. units.*

Remark.—Many equations in magnetism involve the factor b when pole strength, magnetic field intensity, etc., are expressed in Faraday units, in the same way that many equations in electrostatics involve this factor when electric charge, electric field intensity, etc., are expressed in the old c.g.s. units.

TABLE.

Relative values of units.

$$v = 3 \times 10^{10} \frac{\text{cm}}{\text{sec}}$$

One Faraday unit charge $= \frac{1}{v}$ c.g.s. units charge $= \frac{10}{v}$ coulombs.

One Faraday unit current $= \frac{1}{v}$ c.g.s. units current $= \frac{10}{v}$ amperes.

One Faraday unit magnetic field $= \frac{1}{v}$ c.g.s. units magnetic field.

One Faraday unit electromotive force $= v$ c.g.s. units electromotive force $= \frac{v}{10^8}$ volts.

One Faraday unit electric field $= v$ c.g.s. units electric field $= \frac{v}{10^8}$ volts per centimeter

One Faraday unit magnet pole $= v$ c.g.s. units magnet pole.

One Faraday unit capacity $= \frac{1}{v^2}$ c.g.s. units capacity $= \frac{10^9}{v^2}$ farads.

One Faraday unit resistance $= v^2$ c.g.s. units resistance $= \frac{v^2}{10^9}$ ohms.

Remark.—Writing $\frac{1}{4\pi}$ for b in equations (90) and (91) we have

$$C_{\text{Faraday units}} = \frac{1}{4\pi}\frac{a}{x} \qquad (105)$$

$$C_{\text{Faraday units}} = \frac{1}{4\pi}\frac{ka}{x} \qquad (106)$$

in which a is the area of one plate of a condenser in square centimeters, x is the distance between the plates in centimeters, and k is the inductivity of the dielectric.

* These units are usually spoken of as the c.g.s. units of the *electromagnetic system*, new units are usually spoken of as the c.g.s. units of the *electrostatic system*. system is the older of the two, although here spoken of as a new system.

Inspection of equation (105) shows that C is expressed in centimeters (square centimeters divided by centimeters) and the Faraday unit of capacity is called the *centimeter*.

178. Electric field due to a concentrated charge.—The intensity of the electric field at a distance r centimeters from a charge q is

$$f = \frac{q}{r^2} \qquad (107)$$

f and q being expressed in Faraday units.

Proof.—Describe a sphere of radius r with its center at the concentrated charge q. The electric field at the surface of this sphere is everywhere normal to the surface and of the same intensity, f, so that the electric flux across the surface of the sphere is $4\pi r^2 \times f$, $4\pi r^2$ being the area of the sphere; but the flux through the sphere is equal to $4\pi q$, according to equation (104), so that

$$4\pi r^2 f = 4\pi q \text{ or } f = \frac{q}{r^2}. \qquad \text{Q. E. D.}$$

Remark.—Equation (107) also expresses the electric field intensity at a distance r from the center of a sphere upon which a charge q is uniformly distributed.

179. Electrostatic attraction and repulsion of concentrated charges.—Consider a concentrated charge q_1. At a point p distant r from q_1 the electric field produced by q_1 has the intensity $f = \frac{q_1}{r^2}$ according to equation (107). If another concentrated charge, q_2, is placed at the point p it will be acted upon by the force $F = fq_2$ according to equation (98), or, since $f = \frac{q_1}{r^2}$ we have

$$F = \frac{q_1 q_2}{r^2} \qquad (108)$$

in which F is the force with which the two charges q_1 and q_2 repel each other, and r is the distance between the charges.

When q_1 and q_2 are both positive or both negative they repel each other; when one is positive and the other is negative they attract each other.

The fact expressed by equation (108), namely, that two charges attract or repel each other with a force which is inversely proportional to the square of the distance between them, was discovered experimentally * by Coulomb and is called *Coulomb's law of electrostatic attraction.*

The Faraday unit of charge.—The relation expressed by equation (108) furnishes the simplest and most direct definition of the Faraday unit of electric charge. *This unit of charge is that charge which repels an equal charge at a distance of one centimeter with a force of one dyne.*

180. Electric potential at a point, p, distant r from a concentrated charge, q.—We may choose as our region of zero potential (see Art. 160) the region infinitely distant from a given concentrated charge. Then the potential at the given point, p, is equal to the electromotive force along any path from that point, to a point infinitely distant. This path may be chosen, for simplicity, as the straight line passing from q through p. Consider an element Δs of this path distant s from q. The electric field at this element is parallel to the path and, according to equation (107), its intensity is $\frac{q}{s^2}$. Substituting this expression $\frac{q}{s^2}$ for $f \cos e$ in equation (100) we have

$$E = \Sigma \frac{q}{s^2} \cdot \Delta s$$

Integrating this from $s = r$ to $s = \infty$, we have the electromotive force from p to infinity, or the potential at p, which is

$$V = \frac{q}{r} \qquad (109)$$

in which V and q are expressed in Faraday units.

Remark.—The potential at a given point, p, due to a charge distributed in any manner is:

$$V = \Sigma \frac{\Delta q}{r} \qquad (110)$$

That is, the distributed charge is imagined to be divided up into small parts, Δq; each of these small parts is divided by its distance from the given point, p; and these quotients are added together to give V.

181. The spherical condenser.—Consider two concentric metal spheres, A and B, Fig. 116. Let R be the external radius of A, R_1 the internal radius of B, $+q$ the electric charge on A, and $-q$ the electric charge on the inner surface of B. It is required to find the electromotive force between A and B in terms of R, R_1 and q.

* Coulomb's law of electrostatic attraction here appears as a derived result. The derivation is not, however, rigorous inasmuch as Gauss's theorem [Equation ... has not been rigorously proven.

Describe a spherical surface, S, of radius s concentric with A and B, as shown in the figure. The electrical field at this spherical surface is normal to the surface at each point and everywhere of the same intensity f. Therefore, the electric flux through this spherical surface is $f \times 4\pi s^2$ which, by equation (104), is equal to $4\pi q$; so that

$$f = \frac{q}{s^2}$$

This equation shows that the electric field between A and B is just the same as the electric field surrounding a concentrated charge or a uniformly charged sphere standing alone (see Article 178).

Fig. 116.

Choose a radial straight line passing from A to B as the path over which the summation of equation (100) is to be extended. Substitute in this equation $\frac{q}{s^2}$ for $f \cos e$, and integrate from $s = R$ to $s = R_1$. This gives:

$$E = q\left(\frac{1}{R} - \frac{1}{R_{,}}\right)$$

or

$$q = \left(\frac{1}{\frac{1}{R} - \frac{1}{R_{,}}}\right)E \qquad (111)$$

in which E is the required electromotive force between A and B.

Comparing equation (111) with (89) we see that

$$C = \frac{1}{\frac{1}{R} - \frac{1}{R_{,}}} \qquad (112)$$

in which C is the electrostatic capacity, in Faraday units, of the condenser formed by the two concentric spheres shown in Fig. 116.

Remark.—When $R_{,}$ in equation (112) is very large the value of C approaches

$$C = R$$

so that the electrostatic capacity of a sphere, which is at a great distance from all other bodies, is equal to the radius of the sphere.

Remark.—If the region between the spheres in Fig. 116 is filled with a dielectric of which the inductivity is k then the capacity is increased k times.

182. The cylindrical condenser.—Consider two coaxial metal cylinders, A and B, Fig. 117. Let R be the external radius of A, $R_{,}$ the internal radius of B, $+q$ the charge on A, and $-q$ the charge on the inner surface of B. Let l, the length of the cylinders, be very large compared to $R_{\prime} - R$. It is required to find the electromotive force between A and B in terms of R, $R_{\prime}$, l and q.

Describe a cylindrical surface of radius s coaxial with A and B as shown in the figure. The electric field at this cylindrical surface is normal to the surface at each point and everywhere of the same intensity, f. Therefore, the electric flux through the cylindrical surface is $f \times 2\pi sl$, which, by equation (104), is equal to $4\pi q$ so that

$$f = \frac{2q}{sl}$$

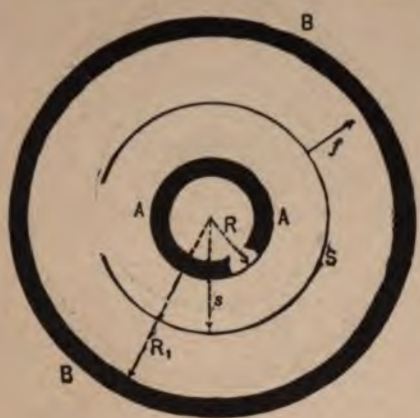


Fig. 117.

Choose a radial straight line passing from A to B as the path over which the summation of equation (100) is to be extended. Substitute in this equation $\frac{2q}{sl}$ for $f \cos e$, and integrate from $s = R$ to $s = R_{\prime}$.

This gives

$$E = \frac{2q}{l} (\log_e R_{\prime} - \log_e R)$$

or

$$q = \frac{l}{2(\log_e R_{\prime} - \log_e R)} \cdot E \tag{113}$$

Comparing equation (113) with equation (89) we see that

$$C = \frac{l}{2(\log_e R_{\prime} - \log_e R)} \tag{114}$$

in which C is the electrostatic capacity in Faraday units of the condenser formed by the two coaxial cylinders in Fig. 117.

183. Electrostatic attraction of parallel plates.—Consider two parallel metal plates each of area a, at a distance x apart, with air between, and charged by a battery of electromotive force E connected between the plates. Then substituting the value of C (in Faraday units) from equation (105) in equation (94) we have

$$W = \frac{2\pi q^2}{a} \cdot x \tag{115}$$

in which W is the energy of the charged condenser.

If the distance x between the plates is increased by the amount *while the plates are entirely insulated so that q cannot change,* the battery being disconnected, the energy of the condenser will increase by the amount

$$\Delta W = \frac{2\pi q^2}{a} \cdot \Delta x$$

and this increase of energy is the work done in pulling the plates apart against their electrostatic attraction. Let F be the force of attraction of the plates; then $F \cdot \Delta x$ is the work done in separating them. Substituting this expression for ΔW in the above equation and solving for F we have

$$F = \frac{2\pi q^2}{a} \qquad (116)$$

in which F is the force of attraction of parallel air condenser plates of area a, $+q$ is the charge on one plate, and $-q$ is the charge on the other plate. It is remarkable that the force of attraction is independent of the distance between the plates *for given charge*, the plates being large compared to the distance between them.

By using the value $C = \frac{ka}{4\pi x}$ from equation (106) we get for the energy of the condenser

$$W = \frac{2\pi q^2}{ka} \cdot x$$

and for the force of attraction

$$F = \frac{2\pi q^2}{ka} \qquad (117)$$

Thus, *for given charge*, condenser plates attract less, the greater the inductivity, k, of the intervening dielectric.

Attraction for given electromotive force.—The charge q on the condenser in the above discussion is $q = \frac{a}{4\pi x} \cdot E$ (or $q = \frac{ka}{4\pi x} \cdot E$). Substituting this value of q in equation (116) [or in equation (117)] we have

$$F = \frac{aE^2}{8\pi x^2} \qquad (118)$$

and

$$F = \frac{kaE^2}{8\pi x^2} \qquad (119)$$

Therefore the attraction of parallel plates, *for given electromotive force*, is inversely proportional to the square of the distance between them and the attraction is greater, the greater the inductivity, k, of the dielectric.

184. The absolute electrometer is an arrangement for determining the value of an electromotive force, using equation (118), by measuring the force of attraction of parallel metal plates. Fig. 118 shows the essential features of this instrument. A portion (a, Fig. 118) of area a, of one plate of a parallel plate air condenser, is hung from one end of a balance beam, so that the force with which this portion, a, is attracted by the opposite plate, bb, may be counterpoised by weights and thus determined.

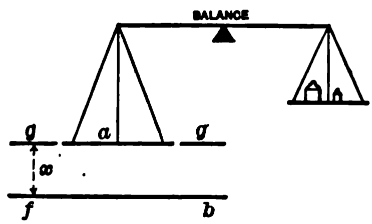


Fig. 118.

The stationary portion, gg, of the upper plate completely surrounds the portion a and is called the *guard ring*. It is to be remembered that equation (118) is true only for plates which are very large, compared to their distance apart, and the function of the guard ring, gg, is to enable this condition to be approximately realized without making a inconveniently large.

Remark.—Equation (118) gives electromotive force in Faraday units when F is in dynes, a in square centimeters, and x in centimeters. One Faraday unit of electromotive force is equal to very nearly 300 volts.

185. The quadrant electrometer.—The absolute electrometer can be used for large electromotive forces only, inasmuch as the force of attraction of the parallel plates is too small to be accurately measured when the electromotive force is small. For example, the attraction of two metal plates, each of 133 square centimeters area, at a distance of one centimeter, is about one ten-thousandth dyne or about one ten-millionth of the weight of a gram for an electromotive force of one volt; while for 10,000 volts the force of attraction about equals the weight of ten grams.

For the measurement of small electromotive forces by electrostatic attraction the quadrant electrometer is used. This instrument is constructed as follows:

A thin plate of metal, *pp*, called the "needle" (Fig. 119) is suspended by a fiber,* which has sufficient torsional rigidity to give to the needle a slight directive tendency. The needle hangs in the interior of a fixed, flat, cylindrical, metal box, which is separated into four quadrants $q_1q_1q_2q_2$. The quadrants q_1q_1 are connected by a wire, and the quadrants q_2q_2 are connected by a

Fig. 119.

wire. The suspended plate has a stiff wire, *w*, projecting downwards and carrying a small metal vane which dips into concentrated H_2SO_4. This acid forms the inner coating of a Leyden jar, *JJ*, which being once charged has sufficient capacity to keep the potential of the needle nearly constant in spite of leakage of charge. The metal vane in the acid serves to dampen the vibrations of the needle; and the acid further serves to keep the air dry inside the case which surrounds the whole instrument.

There are two distinct arrangements of the connections of a quadrant electrometer. In the discussion of these two arrangements we need use only the one principle that *any two charged bodies*

* Ordinarily a bifilar suspension is used.

attract with a force which is proportional to the square of the electromotive force between them, and q_1q_1, q_2q_2 and p, Fig. 119, may be represented by any three bodies as, for example, the three parallel plates q_1q_1, q_2q_2 and p in Figs. 120 and 121.

First arrangement.—In this arrangement the electromotive force, e, to be measured is connected to the needle, p, and to the pair of quadrants, q_2q_2; and the other pair of quadrants, q_1q_1, is connected directly to the needle, as shown in Fig. 120. In this case the force with which p is pulled towards q_1q_1 is zero (zero

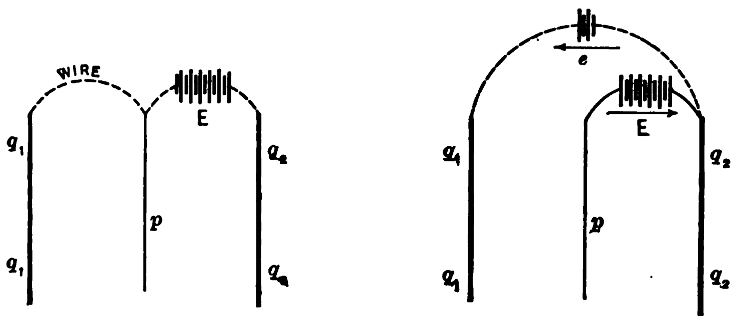


Fig. 120. Fig. 121.

electromotive force between q_1q_1 and p), and the force with which p is pulled towards q_2q_2 is proportional to e^2, and the deflection, d, of the needle is sensibly proportional to e^2, or e is proportional to $\sqrt{d}$, so that

$$e = k\sqrt{d} \tag{120}$$

This arrangement of the quadrant electrometer is always used for measuring the electromotive force of an alternating current dynamo.

Second arrangement.—In this arrangement the electromotive force, e, to be measured is connected to the quadrants, q_1q_1 and q_2q_2, and a large auxiliary electromotive force, E, is connected between q_2q_2 and p, as shown in Fig. 121.

The force with which the needle, p, is pulled towards q_2q_2 is $k'E^2$ (proportional to E^2), and the force with which the needle is pulled towards q_1q_1 is $k'(e + E)^2$, since the electromotive force between p and q_1q_1 is $e+E$. Therefore, the net force pulling p towards q_1q_1 is $k'(e+E)^2 - k'E^2$ or $2k'Ee + k'e^2$; but $k'e^2$ is negligible in comparison $2k'Ee$, since E is very much larger than e, so that the deflecting force, and therefore the deflection, d, is sensibly proportional to Ee, or to e, since E is kept constant, so that

$$e = kd \tag{121}$$

The reduction factor, k, in equation (120) is, of course, different in value from the reduction factor, k, in equation (121).

186. Energy of the electric field.—The whole of the energy of an electric charge resides in the surrounding dielectric by virtue of the electrical stress or electric field.

The energy *per unit volume* of dielectric is

$$W = \frac{k}{8\pi} f^2 \tag{122}$$

in which f is the intensity of the electrical field or stress, and k is the inductivity of the dielectric.

Proof.—Substituting the value of C from equation (106) in equation (96) we have for the energy of a charged condenser:

$$W = \frac{axk}{8\pi} \cdot \frac{E^2}{x^2}$$

but ax is the volume of the dielectric, and $\frac{E}{x}$ is the intensity of the electrical field or stress between the two plates so that equation (122) results at once.

Tension along the lines of force in an electric field.—According to equation (119), the total attraction of two parallel plates is

$$F = \frac{ak}{8\pi} \cdot \frac{E^2}{x^2}$$

Therefore, the pull per unit area is $\frac{k}{8\pi} \frac{E^2}{x^2}$ *and this pull must be due to a state of tension of the dielectric between the plates* so that the tension of the dielectric *per unit area* is

$$F = \frac{k}{8\pi} \cdot f^2 \tag{123}$$

in which f has been written for $\frac{E}{x}$.

CHAPTER XIII.

THE PHENOMENA OF THE ELECTRIC DISCHARGE.

187. Convective discharge and disruptive discharge.—Consider the positive and negative charges at the two ends of a bundle of lines of force. In order that these charges may disappear it is necessary that the lines of force be annihilated. This may occur by the charged surfaces moving towards each other until they are coincident or by the breaking down of the mechanism, the dielectric, which sustains the electric stress which constitutes the electric field. In the former case we have what is known as *convective discharge*, in the latter case we have what is called *disruptive discharge*. Convective discharge is to some extent analogous to the relieving of a stretched rubber band by allowing its ends to move towards each other, thus shortening the band. Disruptive discharge is to some extent analogous to the relief of a stretched rubber band by rupture.

188. Convective discharge.—The transfer of charge by a moving ball, as described in Article 158, is convective discharge. The moving ball gathers in the ends of a bundle of lines of force where it touches one plate and then moves across to the other plate shortening the lines of force until they disappear. Figs. 122, 123, 124, and 125 show the successive aspects of the electric field while the ball is moving once across from plate to plate.

Remark.—When an electric current flows through an electrolyte, the transfer of a charge through the electrolyte is supposed to be accomplished by the movement of charged atoms or arged molecules called ions (see Chapter V. on Electrolysis).

Likewise the flow of electric current through rarefied gases is supposed to be accomplished by the movement of ions.

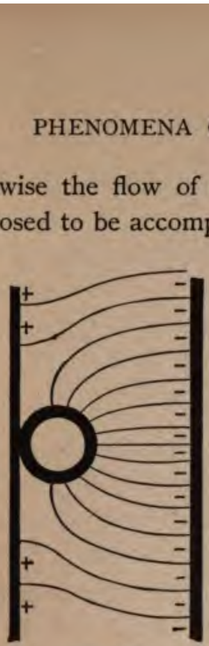


Fig. 122.

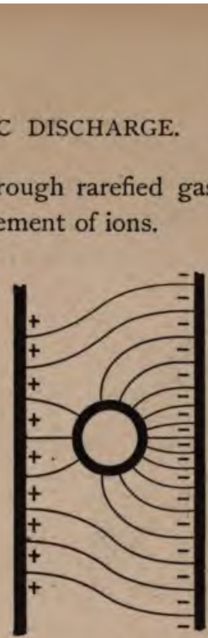


Fig. 123.

189. Disruptive discharge. The electric spark.—Let A and B, Fig. 126, be two conductors, surrounded by any dielectric, on

Fig. 124.

Fig. 125.

which charge is being collected, for example, from an electric machine. The electric stress in the dielectric between A and B becomes more and more intense, as the charges increase, until an *electric spark* is formed between A and B. The duration of the spark is extremely short, and immediately after it A and B are found to be almost if not entirely discharged.

The electric spark shows much the same characteristics in all homogeneous dielectrics, in glass, in oil, in air, or in any gas not too highly rarefied. In a solid dielectric the path of the spark is marked by a line along which the material of the dielectric has been reduced to impalpable powder.

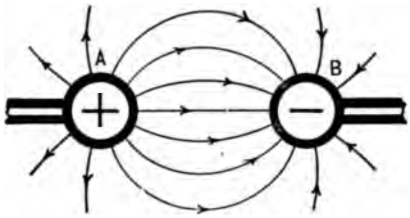


Fig. 126.

190. Electric strength of dielectrics.—The ability of a dielectric to withstand electric stress or electric field, is called the *electric strength* of the dielectric. The electric strength of a dielectric is measured by the intensity of the electric field, ordinarily expressed in volts per centimeter, which is just sufficient to rupture the dielectric.

In a case such as is exhibited in Fig. 126, the electric field is nonhomogeneous, being of different intensities at different points, and the length of the spark is consequently not related in a simple manner to the difference of potential between A and B. Consider, however, two charged bodies, A and B, Fig. 127, having flat portions facing each other. If the corners of A and B are rounded, as shown, the electric field will at no point be more intense than it is in the region between the plates where the field is homogeneous. In this region the field intensity is $f = \frac{E}{x}$ according to equation (99), where E is the electromotive force or potential difference between the plates and x is the distance between the flat surfaces. If charge is collected on A and B until a spark is formed and the corresponding value of E measured, then $f = \frac{E}{x}$ is the electric strength of the dielectric between A and B.

The least roughness of the surfaces of A and B, or clinging floating particles of dust, produce great variations in the value for which a spark is formed. The action of these irregularities of surface, and of floating particles is clearly shown, but somewhat exaggerated, by Fig. 128, in which a floating particle and a minute point projecting from the plate B are represented.

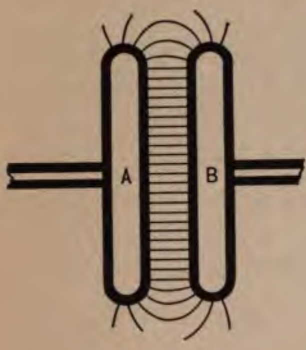


Fig. 127.

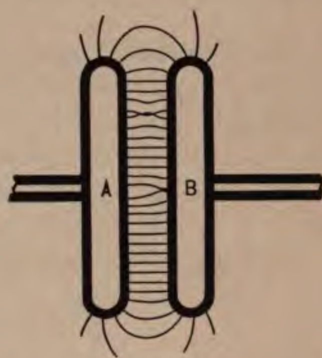


Fig. 128.

The intensity of the field near the point and near the ends of the particle is much greater than the *average* intensity, $\frac{E}{x}$, between A and B and the dielectric will begin to give way at these places when the field intensity there reaches the breaking value.

The following table of the dielectric strength of various substances is from the measurements of Macfarlane and Pierce.*

TABLE.

Dielectric strength of various media.

MEDIUM.	STRENGTH IN VOLTS PER CENTIMETER.	MEDIUM.	STRENGTH IN VOLTS PER CENTIMETER.
Oil of turpentine	94,000	Beeswaxed paper . . .	540,000
Paraffine oil	87,000	Air (thickness 5 cm.) .	23,800
Olive oil	82,000	CO_2 "	22,700
Paraffine (melted) . . .	56,000	O "	22,200
Kerosene oil	50,000	H "	15,100
Paraffine (solid)	130,000	Coal gas "	22,300
Paraffined paper	360,000		

* *Physical Review*, Vol. I., p. 165.

Remark.—The electric strength of a gas seems to be abnormally great near the surface of the negatively charged body, and the electric strength of a gas, determined as above, depends upon the thickness of the layer of the gas.

191. The spark gauge.—The electromotive force necessary to produce a spark between two polished metal balls, or disks of a given size, in air varies in a definite manner with the distance between them. If the electromotive forces required for different distances be once determined by observation, then any electromotive force may be determined by measuring its sparking distance between the given balls or plates. The arrangement for making this measurement is called a spark gauge or *spark micrometer.* The spark micrometer is adapted only to high electromotive forces and the results obtained by it are subject to large errors.

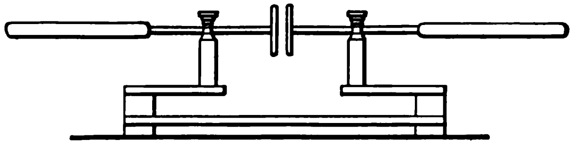


Fig. 129.

In Fig. 129 is shown a form of spark gauge employed by Steinmetz * in his investigation of the laws of sparking distance.

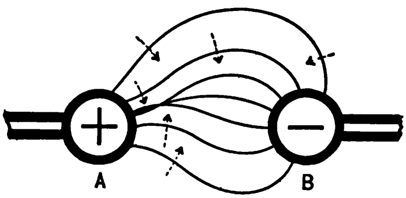


Fig. 130.

192. Progress of the electric spark. Electric oscillations.—Let A and B, Fig. 130, be two metal balls upon which electric charge has accumulated until the intensity of the electric field has reached the breaking point for the intervening dielectric. A *rupture* of the dielectric starts in the region of greatest electric stress,† as

* Steinmetz, *Transactions* of the American Institute of Electrical Engineers, Vol. X.

† This rupture always starts, in air, at the surface of the positively charged body, unless the surface of the other body is much more sharply curved.

indicated by the short, thick line projecting from the surface of *A* in the figure. Along the line of this rupture the dielectric is a good conductor, and the lines of force on all sides move sidewise into the rupture as indicated by the arrows, producing a greatly intensified electric field at the end of the rupture, so that the rupture extends further and further, until it reaches *B*.

This passage of an electric rupture, or electric spark, through a region in which the intensity of the electric field is much below the breaking value for the dielectric, is analogous to the following : A pane of glass is slightly bent and then scratched near one edge so as to start a crack. The effect of this crack is to greatly intensify the stress in the glass at the end of the crack, and the crack therefore quickly runs across the pane.

When the electric rupture has extended itself across from *A* to *B* a conducting line is established from *A* to *B* and all the lines of force, emanating from *A* and *B*, move sidewise into this conducting line and disappear, and the charges on *A* and *B* disappear at the same time.

The disappearance of the charges on *A* and *B* constitutes an electric current along the rupture. This electric current, because of its momentum, persists in flowing after the charges have wholly disappeared and recharges *A* and *B* in a reversed sense.

The reversed charge on *A* and *B* then surges back along the rupture as a reversed current, which by its momentum again charges *A* and *B* as at first, and so on until the energy of the initial charge is dissipated. These back-and-forth surgings of the electric charge are called *electric oscillations*, and the arrangement along which the charge surges back and forth is called an *electric oscillator*.

The dissipation of energy, above mentioned, is due in part to the generation of heat along the line of the rupture in the dielectric, and in part to the fact that electric oscillations produce *electric waves* which travel outwards from *A* and *B* and carry a considerable amount of energy with them.

The oscillatory character of the disruptive discharge is easily shown by causing the image of an electric spark to fall upon a very rapidly moving photographic plate, or by viewing a spark in a very rapidly rotating mirror, or by photographing the image from

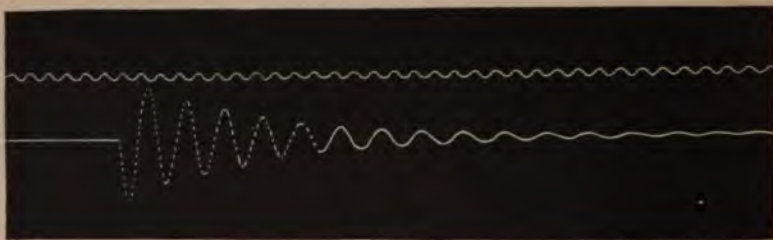


Fig. 131.

such a mirror. The time of a single electric oscillation is in some cases as short as a hundred-millionth of a second or even shorter.

When the disruptive discharge from a large condenser is made to pass through a large coil of wire the oscillations are so slow that the spark emits a shrill musical note of short duration very like the sound produced by striking a steel anvil.

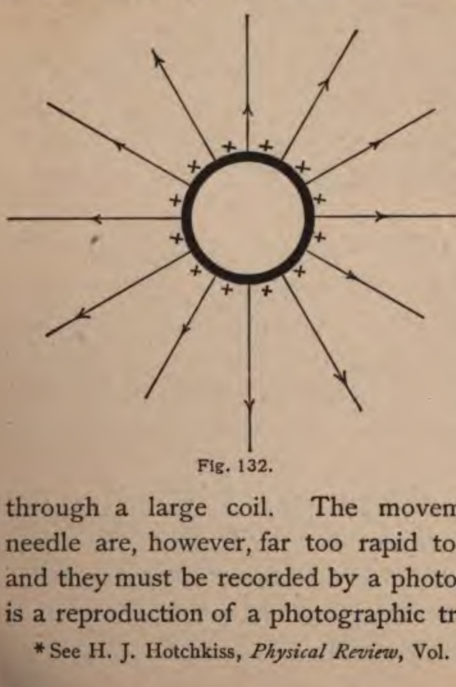


Fig. 132.

By means of a specially constructed galvanometer,* having an extremely light needle, it is possible to follow the fluctuations of the electric current which is produced when a large condenser is discharged through a large coil. The movements of the galvanometer needle are, however, far too rapid to be followed with the eye, and they must be recorded by a photographic tracing. Fig. 131 is a reproduction of a photographic tracing made in this way by

* See H. J. Hotchkiss, *Physical Review*, Vol. II.

Mr. F. E. Millis. The sinuous curve of small amplitude is a time-marking curve, obtained by means of a mirror mounted on a tuning-fork.

193. The brush discharge.—The discharge in air from an isolated conductor, which is charged up to the limit set by the electric strength of the air is, in some respects, different in character from the spark discharge between two oppositely charged conductors which are not too far apart.

In this case, the lines of stress, before the rupture starts, diverge, as shown in Fig. 132, the intensity of the field growing less and less at greater and greater distances from the conductor. The rupture, starting from the surface of the conductor, very soon extends into the region where the field was originally much less intense than at the surface. Such lines of force as have moved sidewise into the fracture and have partially (*i. e.*, through a portion of their length) broken down, now radiate in a widely divergent bundle from the end of the fracture, as shown in Fig. 133. (Compare Fig. 133 with Fig. 130.) The result is that the fracture divides into many branches, which penetrate into the surrounding air in the form of a *brush*. The brush discharge is formed most readily in a region where the lines of electric stress are widely divergent, as on pointed projections of a charged conductor. For some reason, the *brush* forms more readily from a positively charged conductor than from one which is negatively electrified. The positive brush is very different in character from the brush on a negatively charged conductor.

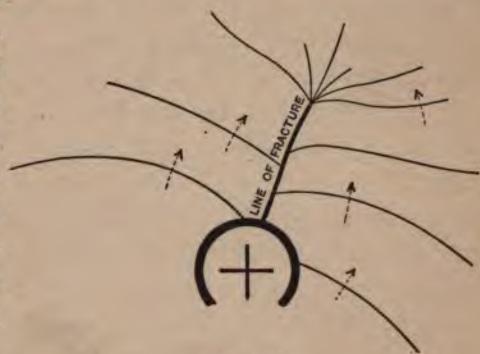


Fig. 133.

194. The electric discharge from metallic points.—A metal rod, *P*, Fig. 134, with a sharp point, is kept charged by connecting it to an electric machine. In the neighborhood of the point the intensity of the electric field is very intense and the surrounding air breaks down for some distance out from the point. Fresh lines of force gather on the point as charge is supplied from the electric machine, and these lines of force, in their turn, break down near the point and so on.

Fig. 134.

This effect of a sharp metal point is so marked that it is almost impossible to keep a perceptible amount of charge upon a pointed conductor. Consider the mass of air surrounding the point, in Fig. 134, in which the lines of force have broken down. *Lines of force pass out from this mass of air to the right, in the figure*, and these lines of force pull the mass of air away from the point in the form of a continuous blast which is quite easily perceptible. At the same time the lines of force which emanate from the metal rod at *b*, Fig. 134, pull the rod to the left. The *electric whirl* is an arrangement of pointed metal rods, bent as shown in Fig. 135 and mounted on a pivot on an insulating stand. When connected to an electric machine, blasts of air issue from the points of the rods and the lines of force issuing from the backs of the rods cause the arrangement to whirl.


Fig. 135.

195. Discharge by hot air or gas.—A hot gas is electrically very weak, and if the lines of force from a charged body pass through such a gas they break down. If the mass of hot gas is at a distance from the charged body, the lines of force near the body will be left intact and also the charge on the body, the mass of hot gas becoming positively charged on one side and negatively charged on the other side as shown in Fig. 104. If, however, the mass of hot gas touches the surface of the charged body, the lines of force break down at the surface of the body and leave it discharged. Thus, the most convenient method for discharging a glass rod (which has been rubbed with silk, for example) is to pass the rod quickly through the flame of a Bunsen burner.

196. Chemical effect of the disruptive discharge in gases.—The stress which is sustained by any dielectric in an electric field seems to tend to break up the molecules of the dielectric, and the rupture which occurs at the time of discharge seems to be of the nature of molecular disintegration along the line of the rupture.

The disruptive discharge through mixed gases promotes chemical combination of those gases. Thus the nitrogen and oxygen of the air combine slowly under the action of the electric spark. It was by this means that the inert atmospheric gas, argon, was discovered. Atmospheric air was enclosed in a glass vessel and a torrent of electric sparks passed through it for several hours until the nitrogen was all oxidized, additional oxgyen being of course supplied. The oxides of nitrogen were then removed by chemical means, leaving the argon and other inert constituents such as krypton, neon, etc.

The electric discharge through oxygen (or air) produces ozone. Thus, an electrical machine in operation gives off a peculiar odor due to the formation of ozone. The ordinary biatomic oxygen molecules, O—O, are broken up by the discharge, forming monatomic oxygen, which immediately recombines, forming mostly biatomic oxygen, again, and also a small amount of triatomic oxygen, , or ozone. The *ozonizer* consists of two parallel metal plates which are repeatedly and rapidly charged and discharged while a stream of air passes between them. To prevent the air from breaking down electrically along a simple line of rupture, or spark, a sheet of glass is placed between the metal plates. By this means the air is made to break down electrically throughout, thus greatly increasing the amount of ozone produced. The complete electrical break-down of the air is shown by the diffused luminosity of the air layer.

197. The electric arc.—When the discharge through the air, or any gas, at ordinary pressure or through a vacuum tube, becomes intense enough to heat the gas to a very high temperature, the discharge assumes a flame-like character between the electrodes, which themselves become very hot, and a very considerable current may be made to pass with but a small electromotive force. Such a discharge is called the *electric arc* or the *arc discharge*. Fig. 136, taken from a photograph, represents the arc discharge between two carbon rods in open air. In this case a current of about 10 amperes was flowing, and the electromotive force between the carbons was about 50 volts.

Fig. 136.

The arc discharge through a gas at low pressure is easily shown by taking a tall U-tube of glass, filling it with mercury, and inverting with each leg in a separate cistern. A dynamo being connected, through a rheostat, to these cisterns, the circuit is momentarily completed by inclining the tube until it is filled with mercury, when, upon being brought again into an erect position, the space above the mercury becomes brilliantly luminous, being traversed by a current of four amperes or more, with an electromotive force of about 20 volts between the cisterns.

198. The effect of pressure upon the disruptive discharge through

gases.—The discussion given in the foregoing articles applies, in so far as it deals with gaseous dielectrics, to gases at atmospheric pressure.

The electric discharge through gases at low pressure is usually studied by means of a glass bulb through the walls of which are sealed platinum wires terminating in metal plates called *electrodes*. The electric current enters at one electrode, called the *anode*, and passes out at the other electrode which is called the *cathode*. This bulb, which is called a *vacuum tube*, is filled with the gas to be studied and the pressure of the gas is reduced to any desired extent by exhausting the tube by means of an air pump.

Before exhaustion the discharge through the tube is in the form of a spark as in the open air. When the pressure of the gas in the bulb has been reduced to a few centimeters of mercury the spark widens and becomes nebulous; as the pressure is further reduced a dark region gradually forms around the cathode; beyond this *dark space* is a region, which (for air) gives off a beautiful bluish violet light, called the *negative glow*. The thickness of the dark space measured normally to the surface of the cathode plate, and of the negative glow, both increase as the pressure decreases. Beyond the negative glow and extending to the anode is a luminous region called the *positive column*. This consists of a succession of bright and dark layers called *striæ*. The distance between adjacent striæ increases as the pressure diminishes. These striæ have in most cases an irregular to-and-fro motion along the tube, which often makes it difficult to distinguish them. These effects are exhibited at their best in a vacuum tube in which the pressure has been reduced to a few millimeters of mercury. Such a vacuum tube is called a *Geissler tube*.

When the exhaustion of the vacuum tube is carried further the dark space which surrounds the cathode expands until it fills the entire tube. The glass walls of the tube then show a brilliant green or blue luminescence, according as the tube is made of German glass or lead glass. A slight negative glow may remain in portions of the tube remote from the cathode or near the anode. These effects, first studied by Crookes in England and by Hittorf in Germany, are exhibited at their best in a vacuum tube in which the pressure has been reduced to a few thousandths of a millimeter. Such a vacuum tube is called a *Crookes tube.*

199. Cathode rays.—An object of any kind placed in a Crookes tube is found to cast a sharp shadow (*i. e.*, a spot where the wall is no longer luminescent) upon the wall of the tube, *as if the cathode were the source of rays which proceed in straight*

FIG. 137.

lines until they strike the walls of the tube where they produce luminescence. Fig. 137 shows a common form of Crookes tube for exhibiting this shadow effect.

These *cathode rays* stream out from the cathode in a direction at right angles to its surface at each point. Thus a cathode in the form of a flat plate gives from its face a bundle of parallel cathode rays. A convex cathode gives a bundle of divergent rays, and a concave or cup-shaped cathode gives a bundle of convergent rays, which concentrate near the center of curvature of the cathode plate, and then, if no obstacle prevents, diverge.

An object upon which the cathode rays impinge is heated, it may be to a very high temperature. Many substances emit ᵹht (without being perceptibly hot) when subjected to the ac- ı of the cathode rays. Such substances are said to be *lumi-*

nescent. For example, lead sulphate emits deep violet light; zinc sulphate emits white light; $MgSO_4 + 1\% MnSO_4$ emits deep red light under the action of the cathode rays. The glass walls of the Crookes tube are, as stated above, luminescent. Barium platinocyanide, magnesium platinocyanide, calcium tungstate, and the various other salts which are used for the luminescent screens in studying the Röntgen radiations (see below), show brilliant luminescence under the direct action of the cathode rays.

The cathode rays pass quite readily through thin metal plates, especially through aluminum and other light metals, which are interposed in their path in the Crookes tube. Lenard, by using a Crookes tube, of which a portion of the wall was made of thin sheet aluminum, got the cathode rays to pass through into the outside air. He found the rays capable of traversing twenty centimeters or more of atmospheric air, of exciting luminescence, and of affecting the photographic sensitive plate.

The cathode rays exert a pressure upon an object upon which they impinge. Crookes mounted a small paddle wheel in a tube so arranged that the cathode stream would fall upon the paddles on one side, and cause the wheel to rotate.

The cathode stream is deflected to one side when it passes through a magnetic field, the direction of the stream, the direction of the field, and the direction of the deflection being mutually perpendicular. This is easily shown by placing a horseshoe magnet with its poles on opposite sides of the tube shown in Fig. 137. The shadow of the cross will be thrown up or down according to the arrangement of the magnet.

200. Crookes' theory of the cathode rays.—Crookes, in his study of the electric discharge in high vacua, was led to think of the discharge as taking place by convection. According to his view, molecules of the gas, perhaps dissociated, come into contact with the cathode, are charged negatively, and hurled off at a high velocity. These projected atoms do not often collide with each other because there are so few of them in the tube, but continue their rectilinear motion until they strike some obstacle. Such negatively charged moving atoms would be equivalent to an electric current towards the cathode, and in this way we may explain the deflection of the cathode rays by a magnetic field. The heating action and force action of the cathode rays, and their action in exciting luminescence, seem to be pretty well explained by Crookes' conception.

Recent experiments of J. J. Thomson seem to show that the moving particles which constitute the cathode rays are extremely small, not more than $\frac{1}{1000}$ as large as a hydrogen atom, and that they move at a velocity equal to one-third or more of the velocity of light. Such flying *corpuscles*, as Thomson calls them, might be able, considering their small size and their enormous velocity, to pass through thin sheets of metal, etc., as cathode rays are known to do.

201. The Röntgen rays.—Objects upon which the cathode rays impinge, not only become heated and luminescent (giving off ordinary light, though not necessarily hot) as described above, but, as discovered by Röntgen in 1894, they emit a type of radiant energy which is not reflected or refracted like ordinary light, but passes straight through all substances, being more or less absorbed, according to the density and thickness of the substance.

The rapidly moving corpuscles which constitute the cathode rays, seem, when they strike an obstacle, to give off very abrupt wave pulses in the ether. These wave pulses are related to ordinary light, to red light, for example, very much as the abrupt solitary sound wave produced by the quick snap of an electric spark is related to the sound waves produced by a sustained musical tone of low pitch. Helmholtz pointed out, in 1891, that abrupt wave pulses in the ether would have certain properties, the properties, in fact, which are exhibited by Röntgen rays.

The fluoroscope.—Many substances, such as barium platinocyanide and calcium tungstate, become luminous when acted upon by Röntgen rays. This action is utilized in the fluoroscope, which consists of a cardboard screen covered with a layer of the one or the other of these salts. When the Röntgen ray shadow of an object, such as the hand, falls on this screen, the screen becomes more or less luminous, according to the amount of absorption of the rays by the various parts of the hand and the shadow is thus rendered visible.

The Röntgen rays affect the ordinary photographic plate and it is possible, therefore, to render the shadow of an object per-

Fig. 138.

Fig. 139.

manent by allowing the shadow to fall upon a photographic plate, which is then developed and fixed in the ordinary way. Fig. 138 is a reproduction of such a shadow photograph of the hand.

The focusing tube.—In order that a shadow may be sharply defined, the radiation which produces the shadow must emanate from a very small source. Fig. 139 shows a Crookes tube with a concave cathode, from which the cathode rays converge and strike a small spot on a platinum plate. This small spot is the source of the Röntgen rays. This tube is called the *focusing tube*.

202. The ionization of gases.—Crookes' theory of the cathode rays, namely that these rays consist of moving charged particles or ions, has been extended in a slightly modified form to include a wide variety of electric phenomena. Certain influences, such as high temperature, intense ultraviolet light, Röntgen rays, and very intense electric stress, seem to dissociate the molecules of a gas producing from each molecule one or more pairs of oppositely charged particles, called *ions*. These ions wander freely about and under the action of the electric field they transfer charge. Thus an electromotive force of a few volts produces a perceptible electric current across several inches of air between two metal plates if the air between the plates is ionized by any of the agencies above mentioned.

Atmospheric electricity seems to be due, in part at least, to the ionization of the atmosphere, especially the upper regions of the atmosphere, by the ultraviolet light in the sun's rays. It has been shown by laboratory experimentation that ions serve as nuclei for the condensation of water vapor, the negatively charged ions especially. The result is that during a thunder shower the negatively charged ions in the upper atmosphere become weighted by condensed particles of water and are drawn to the earth by gravity, thus giving the earth a negative charge -hile the upper atmosphere is left positively charged because of : positive ions which are left there. This condensing action the ions upon water vapor is easily shown by bringing a high :ssure steam jet near to a Crookes tube or near to an arc lamp, iich gives off a large quantity of ultraviolet light. When the :am jet is screened by a sheet of glass it is nearly invisible. hen the screen is removed the jet becomes instantly clouded.

·sla's induction coil.—The following type of induction oroduction of oscillatory sparks of high frequency is Tesla.

A helix, *PP*, Fig. 140, of, say, ten to fifteen turns of wire is connected to the terminals, *CD*, of a large condenser, *AB*, with a spark gap at *g*. The condenser, connected to the secondary of a transformer (10,000 to 20,000 volts), is charged until the air gap at *g* breaks down, when the charge of the condenser surges back and forth through the helix *PP* until the energy of the charge is dissipated. A jet of air issues from a nozzle, *J*, blowing away the air which has been heated and ionized by the spark. Then the charge upon the condenser again increases until a fresh discharge occurs. The successive discharges may be as frequent as several hundred per second, and the oscillations of each discharge may be at the rate of several hundred thousand per second.

Another helix, *SS*, of several hundred turns of wire, is arranged with its axis coinciding with the axis of the helix *PP* (not so shown in the figure). The coils *PP* and *SS* constitute the primary and secondary coils of an induction coil or transformer. The rapidly oscillating current in *PP*, due to the discharge of the condenser, induces enormous electromotive forces in *SS*, producing long sparks between the terminals of *SS*. The coils *PP* and *SS* have to be very highly insulated, for which purpose it is usual to place all the coils in an oil-bath.

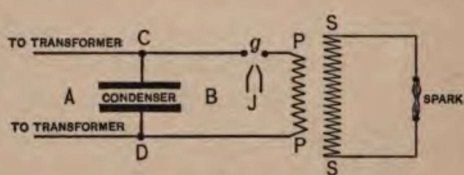


Fig. 140.

A very striking property of the discharge from *SS*, as of any oscillatory discharge of very high frequency, is that it traverses only the layers near the very surface of a wire, or any conductor through which it passes, and it may be in consequence passed through (over) the human body with impunity.

The condenser *AB* should be arranged so that one may change its electrostatic capacity at will. Such a change alters the period of oscillation of the discharge through *PP*, and the operator may thus bring the oscillations of *AB* into unison with the proper oscillations of *SS*, under which condition the action of the arrangement is most intense.

204. Hertz's experiments with electrical waves.—The disruptive discharge is almost instantaneous relief for the electric stress in the immediate neighborhood of the spark. This relief passes out from the line of the discharge as a *wave*. In case the discharge is oscillatory a *train of waves* passes out. The first experimental study of electric waves was made by Hertz, and the following discussion gives a general idea of his experiments.

The oscillator consists of two brass rods, *A* and *B*, Fig. 141, with a spark gap at *g*. These rods are connected with the secondary terminals of an induction coil, as indicated. An impulse from the induction coil charges the rods *A* and *B* (oppositely) more and more until the air-gap, *g*, breaks down, when the discharge surges back and forth along the rods until the energy of the charge is dissipated. This action is repeated with each impulse from the induction coil.

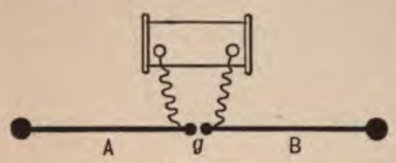


Fig. 141.

The resonator.—The electric waves are detected by means of an arrangement exactly similar to the oscillator, but with a shorter spark gap, and without connections to an induction coil. This arrangement, called the *resonator*, has the same period of oscillation as the oscillator, so that the action upon it of the train of waves from the oscillator is cumulative, causing it to oscillate in sympathy with the oscillator; just as one tuning fork vibrates in sympathy with a similar one which is set vibrating by a hammer blow. The oscillations of the resonator are indicated by minute sparks in its spark gap.

The reflectors.—The waves emanating from the Hertz oscillator are very weak at any considerable distance, and their action upon the resonator is scarcely perceptible. Their action may be greatly intensified by the use of parabolic reflectors. The oscillator and the resonator are placed along the respective focal lines of two parabolic cylinders made of sheet metal. These are shown in vertical and horizontal sections in Fig. 142.

The resonator, *CD* (Fig. 142), is arranged so that the spark gap is behind the mirror, as shown at *g*. It is thus rendered more easily visible.

Reflection of electric waves.—When the oscillator and resonator are arranged as shown in Fig. 142, a very distinct action on the

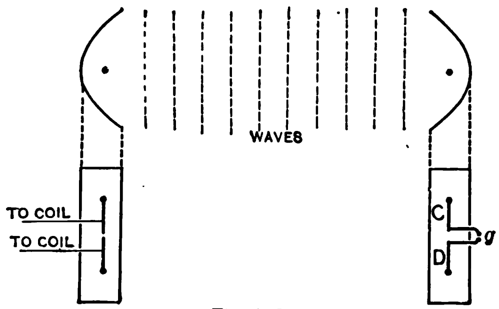


Fig. 142.

resonator is produced when the oscillator is active, the waves emanating from the oscillator being concentrated upon the resonator by the action of the two parabolic reflectors.

When arranged as shown in Fig. 143, *AB* being a plane sheet of metal, and the angles ϕ being equal, a very distinct action on the resonator is likewise produced.

Refraction of electric waves.—When the oscillator and resonator are arranged as

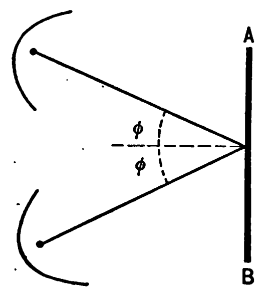


Fig. 143.

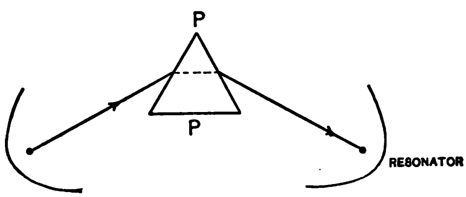


Fig. 144.

shown in Fig. 144, in which *PP* represents a large prism of asphaltum or paraffin, the resonator shows likewise very distinct action.

Polarization of electric waves.—A frame strung with a grating of fine metal wires acts as a good reflector for these electrical waves, when the wires of the grating are parallel to the axis of the oscillator. In this case the grating allows almost no portion of the waves to pass through it. When the wires of the grating are at right angles to the axis of the oscillator the waves pass through it without perceptible diminution in intensity and without perceptible reflection.

Stationary electric waves.—If the oscillator be faced towards a plane metal sheet, *AB*, Fig. 145, the resonator, removed from its parabolic reflector, will be found to show no action near the wall at *p*. As it is moved away from the wall it will become more and more active. Passing a place of maximum activity at *p'*, it will then come into a region of no activity at *p''*, and so on, as represented graphically by the dotted lines.

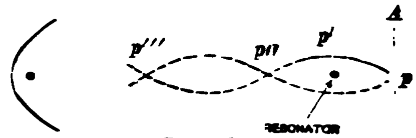


Fig. 145.

The distance *pp'''* is the distance traveled by the electrical waves during one oscillation of the oscillator. If the period τ of the oscillator be determined experimentally, the distance $pp''' \div \tau$ gives the velocity of progression of the electrical waves. This is found to be exactly the velocity of light, viz., $299 \times 10^8 \frac{\text{cm}}{\text{sec}}$. The period of one oscillation of such an oscillator as described above is in the neighborhood of two or three hundred-millionths of a second.

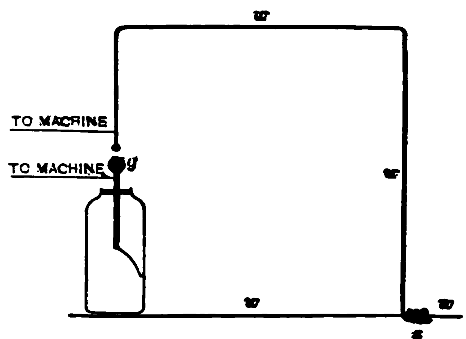


Fig. 146.

Leyden jars as oscillators and resonators.—Two similar Leyden

Jars may be easily arranged as oscillator and resonator respectively as follows:

The jar which is to be used as the oscillator is connected with a loop of wire, *ww*, as shown in Fig. 146, with a spark gap at *g*. Charge collects on the two coatings, the gap, *g*, breaks down, and the charge surges back and forth through the wire *ww*.

The jar which is to be used as a resonator is connected to a similar loop of wire without a spark gap, as shown in Fig. 147. The connection *s* may be slid along *ww* until the apparatus works satisfactorily. The period of oscillation of the resonating jar is then the same as that of the oscillating jar.

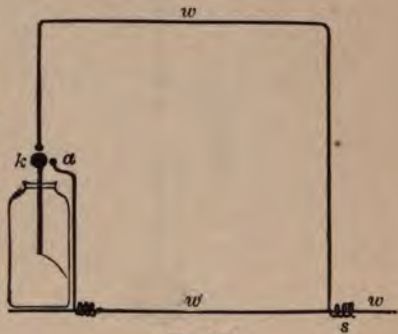


Fig. 147.

In order to detect the sympathetic oscillations of the resonator a wire, *a*, Fig. 147, is connected to the outer tinfoil coating and brought near to the knob, *k*, so as to form a short gap across which a small spark is seen each time a spark passes across *g*, Fig. 146.

The two jars should be placed near together with the wire loops side by side.

205. The coherer.—A light contact between two clean metal rods has ordinarily a very considerable electrical resistance. Branly discovered that the resistance of such a contact is suddenly and greatly reduced when the rods are exposed to electrical waves, in a manner similar to the exposure of the electrical resonator as described in Article 204. It seems that the slight oscillatory current which is forced across the contact by the electrical waves, welds the two rods together at the point of contact.

Fig. 148.

After the cessation of the waves the contact retains its low resistance. A very slight mechanical shock, however, changes the resistance of the contact, bringing it almost instantly to its initial high value.

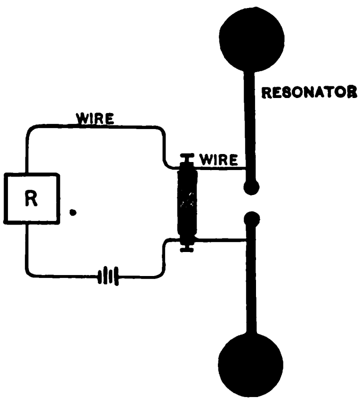


Fig. 149.

This effect is made use of for the detection of electric waves as follows:

A small glass tube, *AB*, Fig. 148, contains a small quantity of clean metal filings lying loosely between the ends of two metal plugs, which project beyond the ends of the glass tube and are provided with binding posts. This arrangement is called a *coherer*.

This coherer is connected across the spark-gap of an electric resonator and to a battery and telegraph relay, *R*, as shown in Fig. 149. Also an electric bell, not shown in the figure, is arranged so that its clapper strikes continuously against the coherer, so as to jar the filings. Under these conditions the electrical resistance of the filings is so great that but little current passes through the relay. When electrical waves strike the resonator, however, the resistance of the filings is suddenly decreased and the battery current increases and operates the relay. When the electric waves cease, the pounding of the bell clapper causes the resistance of the filings to rise again, which lessens the battery current and the relay magnets are no longer excited.

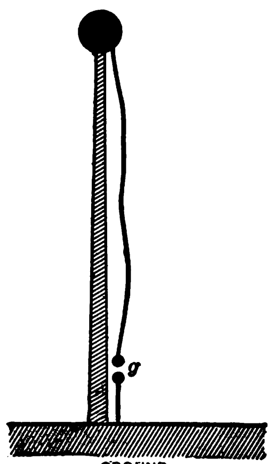


Fig. 150.

206. Wireless telegraphy.—The great sensitiveness of Branly's coherer enables one to detect electrical waves at a distance of a hundred miles or more from the oscillator, so that this process can be used for telegraphic signalling. For this purpose a high staff is erected at each station. At the top of this staff a large insulated metal ball or plate is fixed. From this ball a wire leads down to a spark-gap *g*, Fig. 150, and thence to earth. This metal ball with its connecting wire and spark-gap may be used as an *oscillator* by connecting the terminals of the gap to the secondary terminals of an induction coil; or it may be used as a *resonator*, or *receiver*, in which case a coherer is connected to the terminals of the spark-gap as shown in Fig. 149.

CHAPTER XIV.

SOME PRACTICAL APPLICATIONS OF ELECTRICITY AND MAGNETISM.*

207. The Morse telegraph is an arrangement for signalling between distant stations as follows: An insulated wire leads from one station to the other and back. The ground is generally used instead of a return wire. An electric current from a battery or other source is sent intermittently through this circuit by operating, at one station, a *key* which makes and breaks the circuit. This current excites an electromagnet at the other station, and the armature of this electromagnet either makes a graphical record or produces sound signals. Messages are sent by making use of a code of signals.

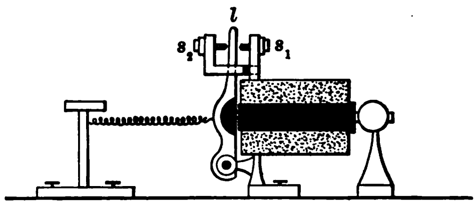


Fig. 151.

Relays and sounders.—A relay consists of an electromagnet, usually wound with many turns of fine wire, which actuates a light lever, *l*, Fig. 151, and this lever is arranged to open and close a separate and distinct electrical circuit, called the *local circuit*, as it moves back and forth between the stops s_1 and s_2. By using a relay, a weak current, only, is needed on the line. This operates the relay and the relay controls the *local circuit* through which a strong current flows and operates the sounder.

* Many practical electric and magnetic appliances, such as dynamos, motors, transformers, induction coils, condensers, lightning arresters, the compensated ship's compass, the magnetic ore separator, the ozonizer, apparatus for wireless telegraphy, and X-ray apparatus, are described in previous chapters. See Index.

The *sounder* consists of an electromagnet, usually wound with coarse wire, which actuates a massive lever (see Fig. 152), arranged to produce audible clicks as it moves back and forth between stops.

The usual arrangement of a telegraph line is as follows: A relay and a key are connected in the circuit with the line wire at each station. When a key at any station is operated, all the relays act simultaneously, and at each station a sounder is actuated by one or two cells of battery under the control of the make and break device of the relay. The battery which furnishes current for the line circuit may be located at any convenient place along the line.

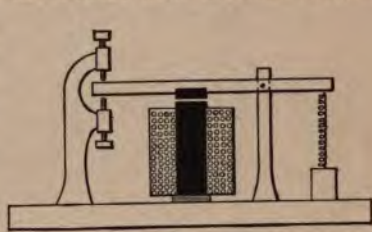


Fig. 152.

The repeater.—On very long telegraph lines there is tendency for the successive pulses of current from a distant sending station to *overlap each other* and become confused as is explained in the discussion of submarine telegraphy. This difficulty is obviated on land lines by the use of the *repeater*.

A long telegraph line is broken up into a number of sections, each having its ground return. A key at the extreme end of the line is operated. At the end of the first section a relay-like device opens and closes the circuit of the second section; at the end of the second section a relay-like device opens and closes the circuit of the third section, and so on. This relay-like device is the repeater. It is so arranged as to operate properly for messages sent in either direction over the line, and it is therefore somewhat complicated.

The Morse recorder consists of an electromagnet which actuates a lever carrying a pencil or stylus under which a strip of paper is moved by clockwork. When the electromagnet is excited, the pencil is pressed against the moving paper and when the exciting current ceases the lever is pulled back by a spring, lifting the pencil from the paper. In this way a record of the signals is obtained.

208. The polarized relay.—The ordinary relay responds to make and break. By using proper tension on the spring which pulls the lever back (see Fig. 151), the lever may be made to respond to *increase and decrease* of current. A quick reversal of current, on the other hand, will not affect the instrument, inasmuch as the lever does not have time to move perceptibly while the current passes through zero value.

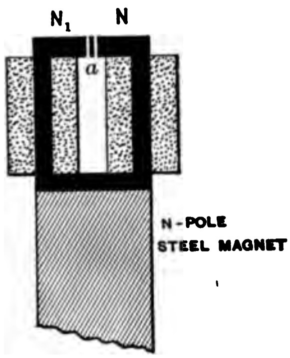


Fig. 153.

The *polarized relay* is so constructed that it responds to *reversals* of current, but does not respond to increase and decrease of current.

An electromagnet, NN_1, Fig. 153, is mounted, as shown, upon one pole of a V-shaped permanent magnet. A light iron lever, *a*, Fig. 154, pivoted at *p*, passes through a slot in the south pole, *SS*, of the permanent magnet, between the poles N and N_1 of the electromagnet, and plays between the stops, p' and p''. This lever, *a*, is magnetized, inasmuch as it bridges over from the south pole, *SS*, of the permanent magnet to the soft iron cores, N and N_1, which stand upon the north pole of the permanent magnet.

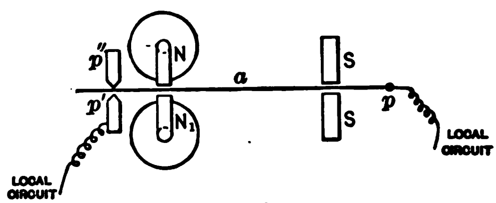


Fig. 154.

When a current flows in a certain direction through the coils of the electromagnet, one of its poles, N_1, for example, becomes a strong north pole, and attracts the lever, *a*. When the current is reversed, the other pole, *N*, becomes a strong north pole, and attracts the lever, *a*. Thus the lever, *a*, is pulled towards

N or N_1, according to the *direction* of the current which flows through the coils of the instrument.

Remark.—The ordinary relay is usually called a *neutral* relay, to distinguish it from the polarized relay.

209. Diplex telegraphy.—The sending of two messages in the same direction over one line wire simultaneously is known as *diplex telegraphy*. This is accomplished as follows: At the sending station are two keys. One of these keys is arranged to vary the *strength* of current in the line (never actually breaking circuit) by throwing a number of batteries in and out of circuit as it is operated. The other key is arranged to reverse the *direction* of the line current as it is operated, the line current being in one direction while this key is down and in the other direction while it is up.

At the receiving station an ordinary relay and a polarized relay are connected in circuit with the line. The ordinary relay responds to the key which varies the strength of the line current, and the polarized relay responds to the key which reverses the line current.

210. Duplex telegraphy.—The sending of two messages in opposite directions over one line simultaneously is known as *duplex telegraphy*. This is accomplished as follows:

Fig. 155 represents the arrangement of apparatus at one station. A similar arrangement is installed at the other station. Let c be the total resistance of the line through the distant station to the ground. The resistances a, b, c, d form a Wheatstone's bridge. When these resistances are so adjusted that $ab = cd$, the key may be pressed

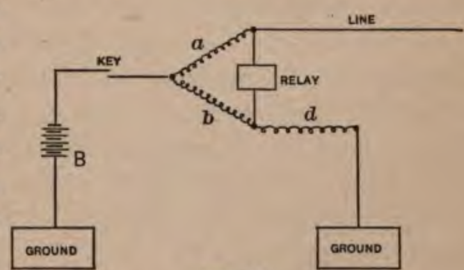


Fig. 155.

without sending current through the relay. When the key is pressed, however, current flows over the line to the other station, and it is easily seen from the figure that a line current coming to a station divides and flows in part through the relay at that station. Therefore the relay at each station responds to the movements of the key at the other station.

211. Quadruplex telegraphy.—The sending of two messages each way over one line simultaneously is known as *quadruplex telegraphy*. This is accomplished by combining the arrangements for diplex and duplex telegraphy. The single key represented in Fig. 155 is replaced by two keys, one for reversing the current, and the other for altering its strength; and the single relay is replaced by two, one an ordinary relay and the other a polarized relay. The polarized relay at each station responds to the reversing key at the other station, and the common relay at each station responds to the key at the other station which alters the strength of the current.

212. Synchronous multiplex telegraphy.—In ordinary telegraphy, where the signals are sent by a key operated by hand, about six or seven current pulses are sent over the line per second and the duration of a single current pulse is never less than about one-tenth of a second. An overhead telegraph line of moderate length, however, can transmit five or six hundred distinct current pulses per second. Therefore a number of operators could send messages over a single wire simultaneously, each to a separate receiving instrument, if means were provided whereby the line could be connected for, say, $\frac{1}{600}$ of a second between the first operator and his receiving instrument, for $\frac{1}{600}$ of a second between the second operator and his receiving instrument and so on in rotation, so that if ten operators were sending messages simultaneously each would have exclusive momentary use of the line sixty times per second. The system of multiplex telegraphy depends upon an arrangement for connecting the two ends of the telegraph wire *synchronously* with the respective sets of telegraphic apparatus and it is therefore called *synchronous multiplex telegraphy*.

The synchronous contact-makers.—The end of the telegraph wire at each station is connected to a revolving metal arm, *A* and *B*, Fig. 156, which touches a number of metal blocks, 1, 2, 3, 4, etc., in succession. The two blocks number one are connected to one set of telegraphic apparatus at the two stations, the two blocks number two are connected to another set of apparatus, and so on, the earth being used as a common return circuit.

The arms, *A* and *B*, rotate at precisely the same speed, so that both arms touch blocks number one at the same instant, blocks number two at the same instant, and so on. The synchronous driving of the two arms is accomplished as follows: Two electric motors drive the arms. *A* and *B*. These motors are adjusted so that when running independently of each other they give approximate synchronism. One or more extra blocks, *r*, Fig. 156, called regulating blocks, are connected to electromagnets, which are arranged to retard the motion of that arm which first touches the block, *r*.

Fig. 156.

Arrangement of telegraphic apparatus.—Fig. 157 shows one set of instruments connected to blocks number one. Similar sets of instruments are connected to the other pairs of blocks. *R* and *R'* are polarized relays, *S* and *S* are switches which stand connected to earth *e*. When a telegram is to be sent the operator throws his switch to *f* and operates his key. The key is connected to two sets of batteries, as shown, so that current flows in one direction over the line when the key is pressed down and in the other direction when the key is raised. These reversals operate the polarized relays, which are not affected by the rapid breaking of the circuit by the rotating arms *A* and *B*.

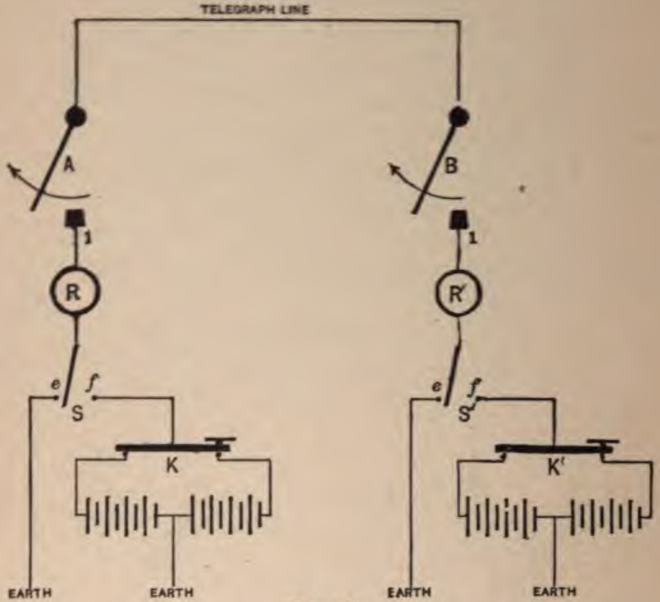


Fig. 157.

213. The printing telegraph is an arrangement by means of which a simple form of typewriter is operated at a distant station from a keyboard at a sending station. The simplest form of printing telegraph is as follows:

Twenty-six equidistant pins are arranged in a helical row around a long metal cylinder. This cylinder is rotated by a small electric motor, and above the cylinder is a bank of twenty-six lettered keys so arranged that when a key is depressed one of the pins comes against it and the cylinder is stopped in a certain position; the next key would stop the cylinder one twenty-sixth of a revolution further, and so on.

Attached to the rotating cylinder is a device for reversing an electric current fifty-two times for each revolution of the cylinder.

These electric currents pass over the telegraph line and through two electromagnets at the receiving station. One of these electromagnets is like a neutral relay with a heavy lever and the other is like a polarized relay with a light lever which oscillates with the rapid reversals of current and actuates an escapement which turns a type-wheel with the twenty-six letters arranged around its periphery. This type-wheel thus turns step by step, keeping pace with the rotating cylinder at the sending station.

When the cylinder at the sending station is stopped by depressing a given key, the A key for example, the current reversing device stops also, a steady current flows over the line, the tongue of the polarized electromagnet stops oscillating, which stops the type-wheel, and the steady current excites the neutral electromagnet, the lever of which pushes a strip of paper against the type-wheel and prints the letter A.

When the key at the sending station is raised the current reversals begin and the type-wheel at the receiving station starts, while at the same time the lever of the neutral electromagnet falls back and actuates a device which moves the strip of paper a step forward for the printing of the next letter.

214. The telautograph is an electrical mechanism by means of which a pencil at a distant station is made to duplicate the movements of a pencil used by a writer at a sending station. The essential features of the telautograph are as follows:

Fig. 158.

The point of the writer's pencil, p, Fig. 158, is connected to two racks, R and r, which engage two pinions, W and w, as shown. Each of these pinions drives two ratchet-like wheels, the teeth of which actuate two make-and-break devices. Two separate and distinct electrical circuits are controlled in this way by each rack and pinion. These four electric circuits pass to the distant station.

Let us consider the two electric circuits which are controlled by the pinion *W*. One of these electric circuits is repeatedly made and broken as the rack *R* moves outwards and the other electrical circuit is repeatedly made and broken as the rack *R* moves inwards.

These two electrical circuits pass to the distant station where they operate two electromagnets which turn the pinion, *W′*, Fig. 159, and move the rack *R′* step by step. One of the electromagnets turns the pinion so as to push the rack *R′* outwards and the other turns the pinion so as to pull the rack *R′* inwards. Thus the rack *R′* duplicates the movements of the rack *R*.

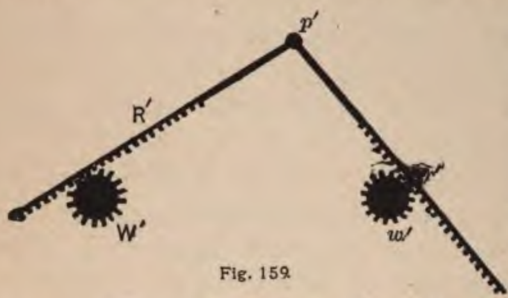


Fig. 159.

The rack *r′* is made to duplicate the movements of the rack *r* in a similar manner, and thus the pencil point *p′* duplicates the movements of the pencil *p*.

215. Submarine telegraphy. — Fig. 160 shows a full-sized sectional view of a submarine cable. The conductor, at the center, consists of a number of strands of copper wire. Surrounding this is an insulating layer of gutta percha and the whole is protected by a covering of tarred hemp and steel wire. The conductor and metal sheath of a cable, together with the intervening insulator, constitute a condenser of large electrostatic capacity.


Fig. 160.

At the instant a battery is connected to a cable a very large current begins to flow into the cable. Most of this current goes to charge the cable, and, as the cable becomes charged, the entering current falls off in value, settling finally to a steady value

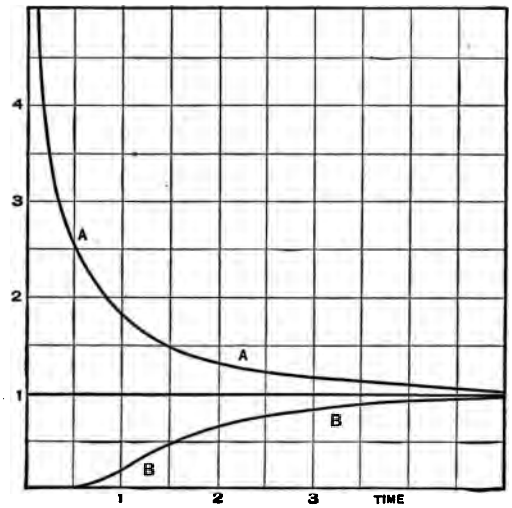


Fig. 161.

which is determined by the resistance of the cable. The ordinates of the curve *A*, Fig. 161, show the successive values of the current which enters a cable from a battery.

At the distant end of the cable an infinitesimal current begins almost at the instant the battery is connected, and as the cable becomes charged this current rises in value until it reaches a steady value nearly equal to the steady value of the entering current. The curve *B*, Fig. 161, shows the growth of current at the distant end of a cable when a battery is connected to the near end.

When the battery is disconnected, the current which enters the cable ceases at once, and the current at the distant end drops slowly to zero as the accumulated charge flows out of the cable.

Distortion of current pulses in a cable.—The curve *a*, Fig. 162, shows the character of the current pulse which enters a cable when a battery is momentarily connected to the cable, and the curve *b* shows the character of the current pulse at the distant end of the cable. The action of a cable in thus altering the

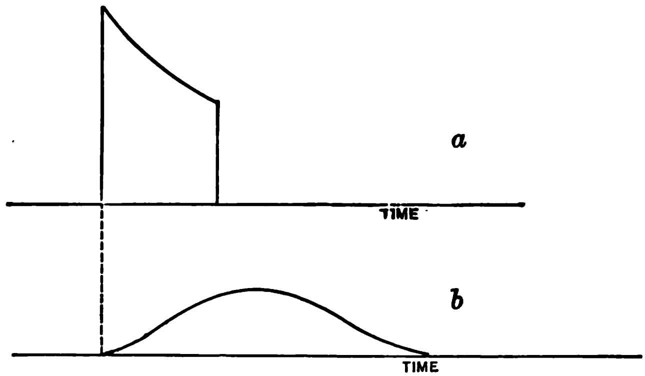


Fig. 162.

character of a current pulse is called *distortion.* Land lines also distort current pulses more or less. The effect of distortion is to cause successive current pulses to overlap at the distant end of a cable, thus setting a limit to the number of distinct signals that can be transmitted in a given time. The curve *a*, Fig. 163, rep-

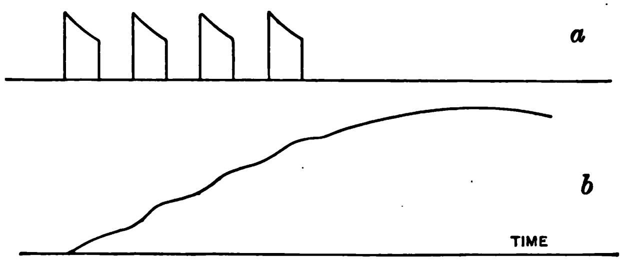


Fig. 163.

resents four short current pulses sent into a cable at one end, and the curve *b* represents the resultant pulse of current that flows out of the other end of the cable. *The receiving instrument in submarine telegraphy is a galvanometer, which is arranged to trace* *the resultant current curve at the receiving end of the cable*, and the separate current pulses that are sent into the cable at the sending end are inferred from the slight kinks in the curve which is traced by the receiving instrument.

Remark.—The distortion of electric current pulses by a submarine cable is analogous to the distortion of pulses of water current by a long thin-walled rubber tube.

216. The syphon recorder is the receiving instrument used in submarine telegraphy. It is a D'Arsonval type galvanometer, the moving coil of which is attached by means of a fine thread to a syphon made of very fine glass tube. This syphon takes ink from a small reservoir and traces an ink line upon a moving paper ribbon. When the galvanometer coil is quiet, a straight line is traced upon the moving paper. When current flows through the galvanometer coil, the coil is deflected, the glass tube is pulled sidewise and a wavy line is traced upon the paper.

It is necessary for the syphon to move sidewise with the utmost freedom, and therefore the tip of the syphon cannot be allowed to rest against the moving paper. This difficulty was overcome in the early form of the syphon recorder by keeping the ink reservoir and syphon highly charged with electricity by means of an influence electric machine, thus causing the ink to issue from the tip of the syphon as a fine jet. In the present form of the recorder the syphon is kept vibrating rapidly against the paper so as to trace a finely dotted line as the paper moves, while, at the same

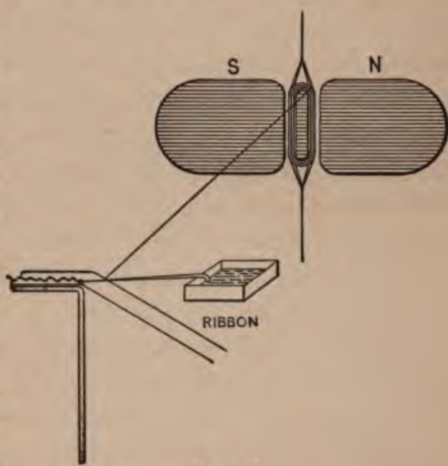


Fig. 164.

time, the sidewise motion of the syphon is not hindered by friction.

Fig. 164 shows the essential parts of a syphon recorder.

217. The telephone consists of a thin sheet-iron diaphragm, *DD*, Fig. 165, very near to which is one end of a steel magnet, *NS*, wound with a coil of fine insulated wire, *cc*.

Action of telephone as transmitter.—The coil *cc* being near the end of *NS*, only a portion of the magnetic flux through the magnet passes through the coil. If *DD* moves nearer to *S*, a greater portion of the magnetic flux through the magnet passes through *cc*, and *vice versa;* but any change of magnetic flux through *cc* induces an electromotive force in *cc* and this in turn produces current in any circuit with which *cc* is connected; therefore *any to-and-fro motion of DD produces currents in the coil cc, which flow in one direction while the diaphragm is moving away from S, and in the other direction while the diaphragm is moving towards S.*

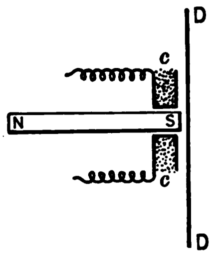


Fig. 165.

Action of telephone as receiver.—If a current passes through *cc* first in one direction and then in the other, the magnet, *NS*, will be correspondingly weakened and strengthened, and the force with which the magnet attracts *DD* will vary accordingly, *causing DD to move to and fro in unison with the currents flowing in cc.*

Consider two telephones connected in circuit. A sound striking the diaphragm of one will cause it to vibrate. This telephone acting as a *transmitter* produces currents which cause a similar vibratory motion of the diaphragm of the other telephone, acting as a *receiver;* and this motion of the diaphragm of the receiver telephone reproduces the original sound.

Remark.—The telephone is an instrument of remarkable sensitiveness. The amount of current necessary to produce an audible vibration of the diaphragm of a receiving telephone depends upon the frequency of the reversals of the current. Ferraris, who has made an investigation of this subject, found for the minimum current necessary to produce an audible sound the following values :*

TABLE.

PITCH IN SINGLE VIBRATIONS PER SECOND.	CURRENT IN AMPERES. (SQUARE ROOT OF MEAN SQUARE.)
269	2.3×10^{-10}
352	1.7×10^{-10}
440	1.0×10^{-10}
523	0.7×10^{-10}
594	0.5×10^{-10}

218. The carbon transmitter.—The currents produced by a telephone acting as a transmitter are very weak, even when the transmitter telephone is exposed to a very loud sound. The *carbon transmitter* is an arrangement by means of which a vibrating diaphragm controls a strong battery current and causes a strong induced current to surge back and forth through the telephone line in unison with the movements of a diaphragm.

The current from a battery passes through the primary of a small induction coil and through a mass of granular carbon, *cc*, Fig. 166. This carbon lies loosely between a carbon block, *B*, and a diaphragm, *DD*. The electrical resistance of the granular carbon varies with the pressure exerted upon it by the vibrating diaphragm, and this causes the battery current to fluctuate. This fluctuating battery current induces currents in the secondary, which pass over the line and affect the receiver telephone at the distant station.

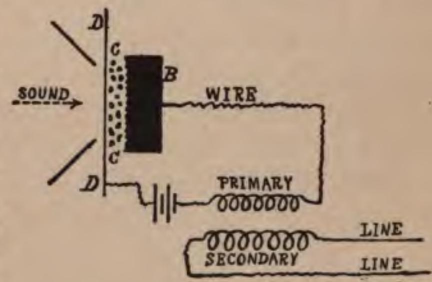


Fig. 166.

219. Long-distance telephony. The distortionless circuit.—The curves in Fig. 167 represent the current pulses sent into a tele-

* Winklemann, Handbuch der Physik, Vol. III. (2), p. 525.

phone line when the indicated vowel sounds are sung into a transmitter by a baritone voice. The current pulses produced by consonant sounds are even more complicated than these, and the clear reproduction of articulate speech by a telephone receiver depends upon the delivery of these current pulses to the receiver telephone without marked distortion.

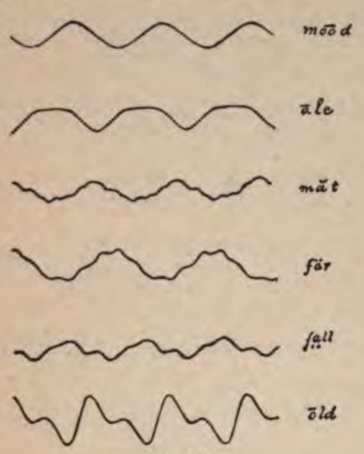


Fig. 167.

Short land lines do not perceptibly distort the current pulses from a telephone transmitter, but with lines a few hundred miles in length the distortion begins to show itself, and telephoning becomes very unsatisfactory over lines a thousand miles or more in length. This trouble is particularly noticeable in large cities where two subscribers may be connected through several miles of cable.

It was first pointed out by Oliver Heaviside that current pulses are not distorted in a circuit of which the resistance, capacity, and inductance are properly proportioned, and Professor M. I. Pupin has shown that the condition pointed out by Heaviside can be realized in practice by introducing inductance coils at intervals along the line, the inductance coils being simply bundles of iron wire wound with insulated copper wire.

220. Electric lighting.—One of the most extended applications of the electric current is in the production of artificial illumination. This is usually accomplished by the heating, to incandescence, of a high-resistance portion of a circuit by the electric current. The high-resistance portion of the circuit, together with its mounting, is called an *electric lamp*. Two types of electric lamp are in general use, namely, the *glow lamp* or *incandescent lamp* and the *arc lamp*.

221. The glow lamp consists of a fine filament of charred vegetable material, upon which a dense deposit of carbon is formed by heating it in the vapor of gasolene. The heating is accomplished by sending electric current through the filament. This filament is mounted in a glass bulb with lead-in wires connecting with the ends of the filament and the air is then exhausted from the bulb. Glow lamps, in commercial use, take from 2½ to 4 watts for each candle power. Thus a 16-candle-power lamp takes in the neighborhood of 50 watts.

Glow lamps are ordinarily connected to the dynamo mains in multiple with each other, as shown in Fig. 168. The dynamo maintains a constant electromotive force between the mains, and each lamp, independently of the others, takes an amount of current equal to the quotient of the electromotive force between the mains divided by the resistance of the lamp.

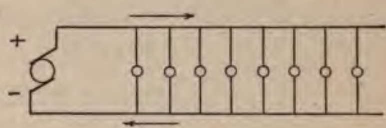


Fig. 168.

Frequently the arrangement shown in Fig. 169, which is called the *three wire system*, is employed. Two similar dynamos are connected to the three wires, and the lamps are arranged in two sets, as shown. The advantage of this system is that, for the same number of lamps, much smaller mains are required than with the arrangement shown in Fig. 168, and a great saving of copper is effected.

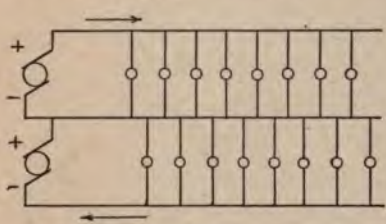


Fig. 169.

222. The arc lamp.—When an electric arc is formed between carbon points, as described in Article 197, the carbon points become intensely hot and give off a very brilliant light. The arc lamp is a mechanism for automatically moving two carbon rods so that a steady electric arc may be maintained between the ends of the rods. There is a great variety of arc lamps, but the following description will serve to give an idea of their action.

The current comes into the lamp and divides as shown in Fig. 170. A very small portion of the current flows through a shunt coil, *B*, without passing through the arc, and the remainder flows through the coil *A*, thence through the arc. An iron rod, *AB*, passing loosely into the coils *A* and *B* is carried upon one end of a lever which turns about the point *p*. The other end of this lever is provided with a clutch, *c*, through which a smooth brass rod, *bb*, passes. This clutch, *c*, is so constructed that it releases the rod, *bb*, when the iron rod, *AB*, is raised, thus allowing the carbons to move closer together.

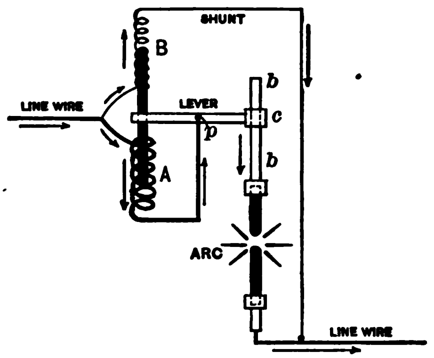


Fig. 170.

Each of the coils, *A* and *B*, acts to pull the rod, *AB*, into itself. A spring attached to the lever is so adjusted that when the arc is burning properly the combined action of this spring and the two coils, *A* and *B*, is to hold the lever in such a position that the clutch grips the brass rod, *bb*. As the arc continues to burn, the carbons are slowly consumed, causing the gap between the carbon tips to widen. This increases the resistance of the arc and causes a greater portion of the current to flow through the shunt coil, *B*, which pulls up the iron rod, moves the lever, releases the clutch, and allows the rod, *bb*, to fall slightly, bringing the carbons again to the proper position.

An arc lamp takes from ⅓ of a watt to one watt for each candle power.

223. The electric furnace is an arrangement for utilizing the heating effect of the electric current for metallurgical purposes or for chemical processes requiring a very high temperature.

In one type of electric furnace the current flows through the material to be heated. Thus, in the manufacture of calcium carbide a very large current is forced through a mixture of powdered coke and lime.

In the manufacture of silicon carbide (carborundum) the current flows through a large core of coke which is surrounded by the material to be heated, namely, a mixture of sand and coke.

For many experimental purposes the electric arc is used, as in the furnace of Moissan. This furnace consists of a small crucible of graphite or magnesia placed immediately beneath a very intense electric arc between horizontal carbon rods. If desired the arc may be thrown down into the crucible by means of a magnet, and the crucible and arc may be enclosed within walls of lime, magnesia or fire brick. Fig. 171 is a sectional view of Moissan's furnace.

Fig. 171.

224. The electrolytic interrupter (Wehnelt).—The primary circuit of an induction coil is usually interrupted by a vibrating reed or spring, which makes and breaks contact between two platinum points. Wehnelt discovered that the sudden generation of oxygen on a small platinum anode in dilute sulphuric acid causes an abrupt stoppage of the electric current. This effect is utilized in the *electrolytic interrupter* as follows:

CC, Fig. 172, is a glass jar filled with dilute sulphuric acid, and provided with two electrodes, *p* and *l*. The anode, *p*, is the tip of a fine platinum wire projecting from a glass tube, and the cathode, *l*, is a large plate of lead. The electromotive force between the mains, which must be 30 volts or more, causes a sudden rush of current through the cell, *CC*, and through the primary of an induction coil. This rush of current generates a layer of oxygen over the platinum tip, which stops the current abruptly. The layer of oxygen then collects as a bubble and rises, leaving the platinum tip again in contact with the acid, when another rush of current takes place, and so on. From two hundred to fifteen hundred interruptions per second are pro-

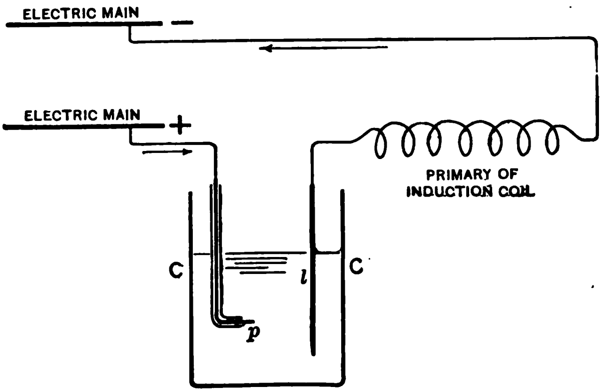


Fig. 172.

duced by this arrangement, according to the size of the platinum tip, the inductance of the circuit and the value of the electromotive force.

Remark.—When the current passes through *CC* in the reverse direction, making the platinum tip the cathode, the tip becomes intensely heated. This intense heating of a small cathode is utilized in the wet process of electric welding as described below.

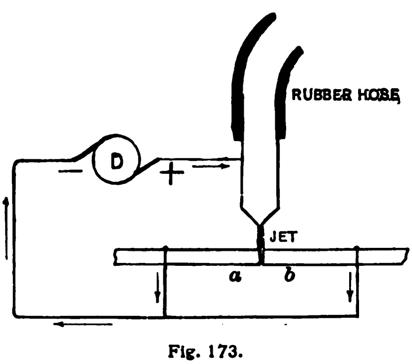


Fig. 173.

225. Electric welding. *Thomson's process.* — The two metal rods to be welded are connected to the terminals of an electric generator and brought into contact with each other. The current flowing across the relatively high resistance contact heats the ends of the rods to the melting temperature, the rods are then pushed slightly together and the weld is complete. Alternating current is generally used in this welding process, a transformer taking current at medium high voltage and supplying a very large current at low voltage to the rods to be welded.

The wet process.—When a direct current generator having an electromotive force of from 200 to 500 volts is connected to an electrolytic cell having a small cathode (the electrode at which the current leaves the cell) the cathode becomes intensely heated. This effect is utilized for welding as follows:

The two rods, *a* and *b*, to be welded are connected to the negative terminal of the direct-current dynamo, *D*, as shown in Fig. 173. The positive terminal of the dynamo is connected to a metal nozzle from which a jet of salt water issues. This jet impinges upon the ends of the two rods and quickly fuses them together.

CHAPTER XV.

THE MECHANICAL CONCEPTIONS OF ELECTRICITY AND MAGNETISM.

226. Preliminary statement.—The hydraulic conception of the electric current, heretofore used, has been admittedly incomplete, inasmuch as it gives no conception of the magnetic effect of the electric current. In a general way one may say that the hydraulic conception of the electric current gives no idea of the relation between *electricity* and *magnetism.*

Maxwell first pointed out an adequate mechanical conception of electricity and magnetism. This conception has been quite fully worked out by Lodge and is outlined in the present chapter. Some inconsistencies arise in the extension of this conception to three dimensions, as has been noticed by Poynting.

227. Fundamental conception.—The ether is to be considered as built up of very small cells of two kinds, *positive* and *negative,* in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact, that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it, and which is proportional to the angle turned.

Fig. 174.

For example, the ether cells may be thought of as small cog wheels with *rubber* teeth, positive cells and negative cells gearing into each other as shown in Fig. 174.

In subsequent figures these cog wheels are represented by plain circles.

228. Conception of the magnetic field.—The ether cells at a point in a magnetic field are to be thought of as rotating about axes which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are geared together.

This rotatory motion of the ether cells is represented in Fig. 175. The magnetic field is perpendicular to the plane of the paper, and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counterclockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

Fig. 175.

229. Conception of the electric field.—The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 176 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Fig. 177 represents two meshes. The downward displacement of the positive cells has distorted these meshes, which are normally square. Since this cell structure of the ether is assumed to be elastic, its distortion, as represented in Figs. 176 and 177, represents potential energy. The amount of potential energy per unit volume is proportional to the square of the displacement. This is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

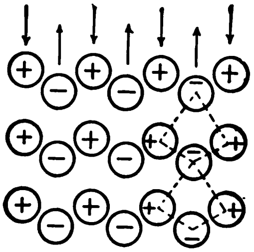


Fig. 176.

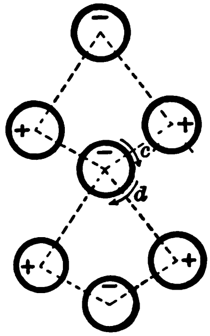


Fig. 177.

Remark.—The two positive cells to the right of the middle cell in Fig. 177 being displaced downwards, may be conceived to exert torques upon the middle cell, as shown by the arrows *c* and *d;* which torques are proportional to the intensity of the electric field, *i. e.*, to the displacements of the cells. The cells to the left exert equal but opposite torques upon the middle cell. *This torque action which accompanies the distortion of the cell structure of the ether is the connecting link between electric field and magnetic field.*

230. The energy stream in the electromagnetic field.—A region in which electric field and magnetic field coexist is called an electromagnetic field. It has been shown by J. H. Poynting * from theoretical considerations that *energy streams through an electromagnetic field in a direction which is at right angles both to the electric field and the magnetic field at each point*, and *that the amount of energy per second which streams across one square centimeter of area is proportional to the product of the electric and magnetic field intensities.* In case the electric and magnetic fields are not at right angles to each other the energy stream is proportional to the product of the intensities of the two fields into the sine of the included angle.

Explanation of energy stream.—Consider three gear wheels, *A*,

* *Philosophical Transactions*, 1884.

B and *C*, Fig. 178. Let *A* and *C* exert equal and opposite torque actions upon *B*. Then, if the wheels are turning, work will be transmitted from *A* to *C*, or from *C* to *A*, according to direction of turning and to direction of torque action, and the rate of transmission of work will be proportional to the product of torque action into speed.

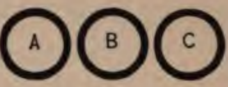


Fig. 178.

Imagine the cells in Fig. 176 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the torque actions between the cells, as explained in Article 229, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The *energy stream*, that is, *energy per unit area per second*, is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both.

231. The electric current.—Consider a wire, *AB*, Fig. 179, along which an electric current is flowing. The magnetic field on opposite sides of *AB* is in opposite directions, so that positive ether cells at *p* and *p'* are rotating in opposite directions, as shown. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of *AB* cannot be due to an ever-increasing ether distortion

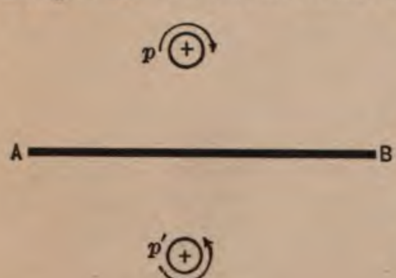


Fig. 179.

(the cells are geared together, as it were), but there must be a *slip* between adjacent cells somewhere between p and p'. This *slip* between adjacent ether cells (positive and negative cells) takes place in the material of the wire, and constitutes the electric current.

232. Established electric currents flow in closed circuits.—Let AB, Fig. 180, be a wire carrying an established electric current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of AB cannot continue without adjacent cells slipping on each other somewhere along any line passing around the end of AB.* That is, established *lines of slip* of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, an increasing ether distortion (electric field) is produced around the end portions of the circuit, which distortion produces (constitutes) electric charge there. Compare Article 235 below.

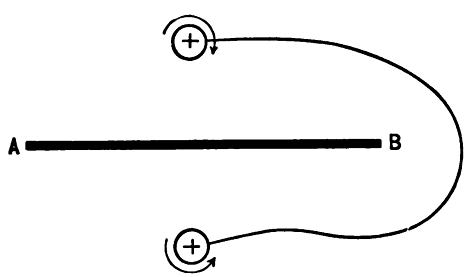


Fig. 180.

233. Flow of energy in the neighborhood of an electric current.—Let Fig. 181 represent the neighborhood of a long wire, AB,

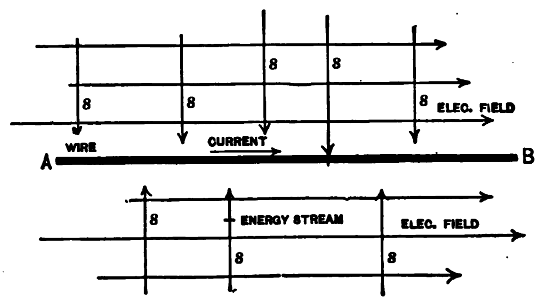


Fig. 181.

* The figure represents a flow in two dimensions.

through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles around the wire. The product of magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for the same current and same intensity of magnetic field, the energy stream is correspondingly intense, making the wire very hot. At points not very near to the wire the electric field is more nearly perpendicular to the wire, especially near the battery or dynamo which is maintaining the current. The energy therefore streams out from the battery or dynamo through the whole region *surrounding* the wires, and the energy stream turns in upon the wire throughout its length.

234. The charge on a condenser and its disappearance when the condenser plates are connected by a wire.—Consider a closed chain of gear wheels, *AB*, Fig. 182. If the gears are allowed to slip at any point, *s*, the gear *f* being held stationary and the gear *e* turned in the direction of the arrow, then the chain of gears will be distorted, as shown in Fig. 183. Conversely, a chain of geared wheels, which by elastic action tend to stand in a row,* will be *relieved* from such a zigzag distortion as shown in Fig.

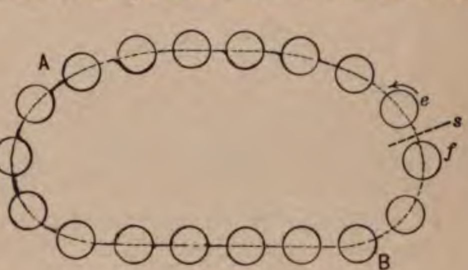


Fig. 182.

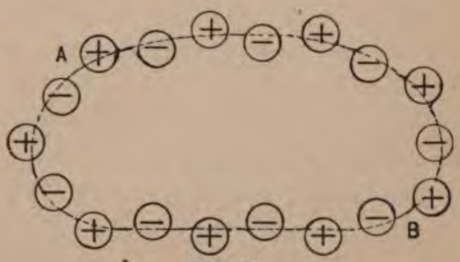


Fig. 183.

* The chains of positive and negative ether cells are thought of as standing in

183, *by permitting the gears to slip anywhere along the chain and the potential energy stored in the distorted chain will be geared towards the place where slipping takes place from both sides of the slip.*

Let A and B, Fig. 184, be two metal plates and let the dotted lines represent closed chains of geared ether cells undistorted. An electric current forced through the wire means the forced slipping of ether cells all along the wire *and each chain of*

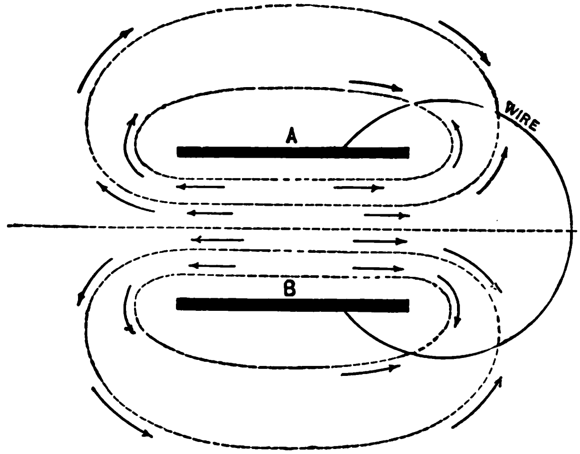


Fig. 184.

geared cells, initially like Fig. 182, becomes distorted like Fig. 183. Throughout the region between A and B the positive ether cells are displaced upwards and the negative cells are displaced downwards, that is, this region becomes an electric field and the plates A and B become oppositely charged.

If the charged plates are connected by a wire, as shown in Fig. 184, each closed chain of geared cells is cut by the wire, slipping begins all along the wire and the energy of each distorted chain of cells is transmitted along the chain, flowing into the wire as indicated by the arrows in the figure.

zigzag rows when undistorted, as shown by the horizontal rows in Figs. 175 and 176. Hereafter the chains of cells are to be thought of as straight (or uniformly curved) when free from distortion, in order that the diagrams may be simpler.

The explanation here given of the entire relief of the electric stress between two plates by the establishment of a conducting line (line of slip) between them, applies to two adjacent oppositely charged bodies of any shape.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the electric energy flows in upon a spark as it does upon a wire carrying current.

The slipping of the ether cells in a conductor is imagined to be opposed very much as if the cells were turning in a viscous liquid.

235. The electric oscillator.—Let *AB*, Fig. 185, be the balls of the oscillator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along the dotted line, which is everywhere perpendicular to the lines of

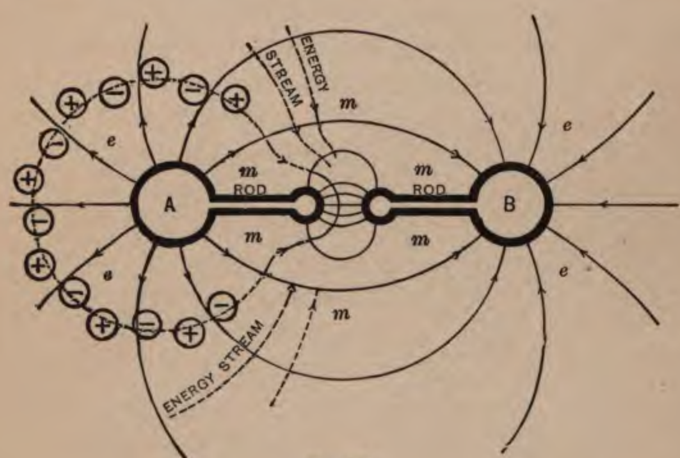


Fig. 185.

force. When *A* is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as explained in Article 234.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted line, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotary motion of the cells between *A* and *B* will have reached a maximum, and on account of their momenta the cells will continue turning, and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the regions *ee*, the ball *A* will be negatively charged, and the ball *B* will be positively charged. This reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

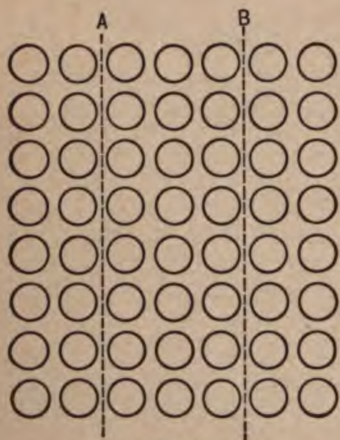


Fig. 186.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from *AB* do not follow the changes promptly. This gives rise to electrical waves, the nature of which at a distance from the vibrator is explained in the following article.

236. Electromagnetic wave. — Imagine the ether cells between the dotted lines *A* and *B*, Fig. 186, to be suddenly set in rotation (only positive cells are shown in the figure). Elastic distortion will be quickly produced between these cells and the stationary cells to the right and left, the effect of which will be to stop the rotation of the cells between A and B and set the adjacent cells in motion. These cells will then transmit their motion to the cells next beyond and so on. Thus, the original disturbance of the layer of cells between A and B will give rise to two waves, one traveling to the right and the other to the left.

Each of these waves consists of a moving layer or region in which the magnetic field and electric field are at right angles to each other, and both are at right angles to the direction of progression of the wave.

237. Stationary electromagnetic waves. Reflection with and without change of phase.—A full understanding of this article requires a previous study of the theory of wave trains.*

Consider the row of cells AB, Fig. 187, along which a train of electromagnetic waves is approaching a conducting wall, W. Imagine the wall to be a perfect conductor, for simplicity; then the wave train is totally reflected, and the reflected train superposed upon the incident train gives a stationary train. The disturbance at any point is the superposition of the disturbances at that point due to the incident and reflected trains respectively. Now at the face of the wall the electric field must be zero (the ether may be imagined to end here, and the freedom of motion of the end cell prevents elastic distortion between that cell and the next), so that the electric field intensities of the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of electric intensity (without change of phase of magnetic intensity).

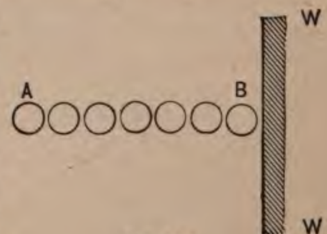


Fig. 187.

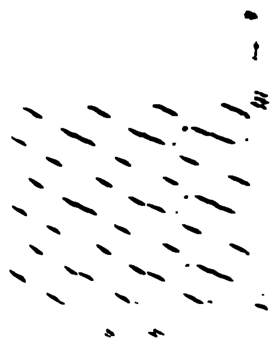
If the wall W be imagined to have infinite magnetic permeability (indefinitely great moments of inertia of ether cells), then the magnetic field intensity at the wall would necessarily be zero, so that the magnetic intensities of the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of magnetic intensity (without change of phase of electric intensity).

In a stationary electromagnetic wave train the energy streams from regions near the nodes (electric) to regions in the vibrating segments (electric) and back again, in a manner very similar to the back-and-forth flow of the energy in the region surrounding an electric oscillator.

238. The dependence of electromotive force upon changing magnetic flux.

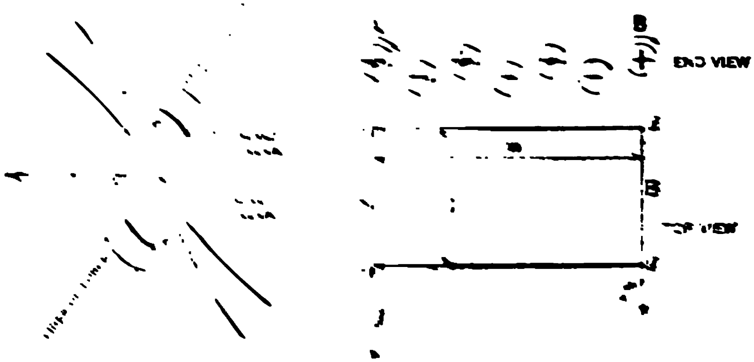
* See Vol. III.

[illegible] —The electromotive force around a closed [illegible] change of the magnetic flux through the curve.



Consider a closed boundary $ABCD$, Fig. 188, [illegible] field parallel to DA and CB. Let [illegible] of the field along CB, and f' [illegible] along DA, and let l be the length of [illegible]. The electromotive force around [illegible] $ABCD$ is $lf' - lf$. Now, the total [illegible] BC on the enclosed cells is [illegible] and to f, and the total torque [illegible] is proportional to l and to f', [illegible] direction. Therefore, the total [illegible] the enclosed cells is proportional [illegible]. The enclosed cells gain angular [illegible] this torque at a rate [illegible] to the torque, and [illegible] so that the *product* [illegible] rate of change of [illegible] force around [illegible] of magnetic field, is equal [illegible].

[illegible] of a conductor moving [illegible] —Consider [illegible] a magnetic field of



[illegible] being a [illegible] the energy of the magnetic [illegible] cannot be geared across A, but an electric [illegible] about the ends of A, as indicated by the lines of force, and the magnetic energy is geared around A, as indicated by the lines marked energy stream.

240. The dependence of magnetomotive force upon changing electric flux. —The magnetomotive force around any closed curve is proportional to the rate of change of the electric flux through the curve.

Consider a distorted chain of cells AB, Fig. 190. The rate at which this distortion is increasing is proportional to the difference in the angular velocities of the cells (intensities f and f' of the magnetic field) at A and B divided by the number of cells in the chain (distance m from A to B). Therefore, the product of the rate of increase of this distortion into the area lm of $cdeg$ (rate of increase of the electric flux through $cdeg$) is proportional to $lf - lf'$, which is the magnetomotive force around $cdeg$.

CHAPTER XVI.

RÉSUMÉ OF ELECTROMAGNETIC THEORY. THE ELECTROMAGNETIC THEORY OF LIGHT.

241. Résumé of electric and magnetic equations.*—In order to obtain a general view of the relations of the various electric and magnetic quantities a number of equations are here collected, and a brief description is given of each.

It is important to distinguish those equations which are *independent*, those which are mere *definitions* and those which are *derived*. Those equations only are independent which formulate independent experimental facts.

242. Independent equations.

$$F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2} \qquad (1)$$

in which F is the force with which two electric charges Q_1 and Q_{11} repel at distance d from each other in a medium of specific inductive capacity k. This equation is derived in Article 179, except that the factor k is not there considered. The effect of specific inductive capacity k is explained in Article 183.

$$F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2} \qquad (2)$$

in which F is the force with which two magnetic poles m_1 and m_{11} repel at distance d from each other in a medium of which the magnetic permeability is μ.

$$F = Qe\dagger \qquad (3)$$

in which F is the force which acts upon a charge Q when placed at a point in an electric field of intensity e. Equation (1) may be written

$$F = Q_1\left(\frac{1}{k}\frac{Q_{11}}{d^2}\right)$$

$$F = mf \qquad (4)$$

in which F is the force which acts upon a magnetic pole of strength m when placed at a point in a magnetic field of intensity f. Equation (2) may be written

$$F = m_1\left(\frac{1}{\mu}\frac{m_{11}}{d^2}\right)$$

* For brevity the equations in this chapter are numbered consecutively, irrespective of the numbers previously assigned to them.

† Heretofore the letter f has been used for intensity of electric field and for intensity of magnetic field alike.

so that putting

$$e \equiv \frac{1}{k} \frac{Q_{11}}{d^2}$$

we have equation (3). However, there are other electrical fields than those which emanate from electric charges

$$\left(= \frac{1}{k} \frac{Q}{d^2}\right)$$

and equation (3) has greater generality than is compatible with its derivation from equation (1), using the definition

$$e \equiv \frac{1}{k} \frac{Q}{d^2}$$

so that putting

$$f \equiv \frac{1}{\mu} \frac{m_{11}}{d^2}$$

we have equation (4). However, there are other magnetic fields than those which emanate from magnetic poles

$$\left(= \frac{1}{\mu} \frac{m}{d^2}\right)$$

and equation (4) has greater generality than is compatible with its derivation from equation (2), using the definition

$$f \equiv \frac{1}{\mu} \frac{m}{d^2}$$

243. Definitions.

$$E \equiv \Sigma e \cdot \cos \varepsilon \cdot \Delta s^{*} \qquad (5)$$

That is, the electromotive force E along any line is *defined* as the line integral of e along that line. (See Article 159.)

$$N \equiv \Sigma ke \cdot \cos \varepsilon \cdot \Delta S \qquad (7)$$

That is, the electric flux N‡ across a surface is *defined* as the surface integral of ke taken over the surface. (See Article 175.)

$$\Omega \equiv \Sigma f \cdot \cos \varepsilon \cdot \Delta s^{*} \qquad (6)$$

That is, the magnetomotive force Ω along any line is *defined* as the line integral of f along that line.†

$$M \equiv \Sigma \mu f \cdot \cos \varepsilon \cdot \Delta S \qquad (8)$$

That is, the magnetic flux M‡ across a surface is defined as the surface integral of μf taken over the surface. (See Article 25.)

244. Derived equations.

$$e = \frac{1}{k} \frac{Q}{d^2} \qquad (9)$$

in which e is the intensity of the electric field at a point distant d from a concentrated charge Q in a medium of specific induction capacity k. It is derived from equations (1) and (3). (See Article 178.)

$$N = 4\pi Q \qquad (11)$$

$$f = \frac{1}{\mu} \frac{m}{d^2} \qquad (10)$$

in which f is the intensity of the magnetic field at a point distant d from a concentrated pole m in a medium of permeability μ. It is derived from equations (2) and (4). (See Article 7.)

$$M = 4\pi m \qquad (12)$$

*The equation $E \equiv \Sigma e \cdot \cos \varepsilon \cdot \Delta s$ reads: E STANDS FOR $\Sigma e \cdot \cos \varepsilon \cdot \Delta s$. Such an equation ranks, of course, as an *independent* equation so far as algebra is concerned. There is, however, not the least physical significance to such an equation.

† Many of the following magnetic equations have not been previously discussed. The proof of these magnetic equations is in each case *identical* to the proof of the corresponding electric equation.

‡ Heretofore the Greek letter Φ has been used for electric flux and for magnetic flux alike.

in which N is the total outward electric flux from a charge Q. It is derived from equations (9) and (7). (See Article 177.)

Electric tube.—Consider a bundle of lines of force in an electric field. This we shall call an *electric tube* for brevity. Let N be the electric flux through the tube and $\pm Q$ the charge at the ends of the tube. Then $N = 4\pi Q$ by equation (11). If the tube is endless, then $\frac{1}{4\pi} N$ is the charge which would be associated with one end of it were it cut across. We shall hereafter speak of the charge associated with one end of an electric tube as the *strength* of the tube.

$$E = \frac{1}{k} \frac{Q}{d} \qquad (13)$$

in which E is the electromotive force from a point distant d from a charge Q, to infinity in a medium of specific inductive capacity k. It is derived from equation (9), using (5). The electromotive force from a given point to infinity, or to any arbitrarily chosen region, is called the *electric potential* at the point when it is independent of the path over which the electromotive force is reckoned. (See Article 180.)

$$W = QE^* \qquad (15)$$

in which W is the work done in moving a charge Q along a line over which the electromotive force is E. It is derived from equation (3), using (5). (See Article 156.)

$$W = \frac{1}{8\pi} NE \qquad (17)$$

or using equation (11)

$$W = \tfrac{1}{2} QE \qquad (19)$$

in which W is the total energy of an electric

in which M is the total outward magnetic flux from a pole m. It is derived from equations (10) and (8). (See Article 26.)

Magnetic tube.—Consider a bundle of lines of force in a magnetic field. This we shall call a *magnetic tube* for brevity. Let M be the magnetic flux through the tube and $\pm m$ the magnetic pole strength at the ends of the tube. Then $M = 4\pi m$ by equation (12). If the tube is endless, then $\frac{1}{4\pi} M$ is the pole strength which would be associated with one end of it were it cut across. We shall hereafter speak of the pole strength associated with one end of a magnetic tube as the *strength* of the tube.

$$\Omega = \frac{1}{\mu} \frac{m}{d} \qquad (14)$$

in which Ω is the magnetomotive force from a point distant d from a pole m to infinity in a medium of permeability μ. It is derived from equation (10), using (6). The magnetomotive force from a given point to infinity, or to any arbitrarily chosen region, is called the *magnetic potential* at the point when it is independent of the path over which the magnetomotive force is reckoned.

$$W = m\Omega^* \qquad (16)$$

in which W is the work done in moving a magnetic pole m along a line over which the magnetomotive force is Ω. It is derived from equation (4), using (6). (See Article 123.)

$$W = \frac{1}{8\pi} M\Omega \qquad (18)$$

or using equation (12)

$$W = \tfrac{1}{2} m\Omega \qquad (20)$$

in which W is the total energy of a mag-

* The electromotive force in equation (15) is understood to be due to *other charges* than Q which is being moved; and the magnetomotive force in equation (16) is understood to be due to something besides the pole m which is being moved.

tube through which the electric flux is N, or at the ends of which are charges $\pm Q$, E being the electromotive force along the tube. Equations (17) and (19) hold for endless electric tubes also. Compare remarks following equation (11). (See Article 157.)

$$W = \frac{k}{8\pi} e^2 \qquad (21)$$

in which W is the energy *per cubic centimeter* at a point in an electric field, e being the intensity of the field at the point, and k the specific inductive capacity of the dielectric. This equation is derived from (17). (See Article 186.)

netic tube through which the magnetic flux is M, or at the ends of which are poles $\pm m$, Ω being the magnetomotive force along the tube. Equations (18) and (20) hold for endless magnetic tubes also. Compare remarks following equation (12).

$$W = \frac{\mu}{8\pi} f^2 \qquad (22)$$

in which W is the energy *per cubic centimeter* at a point in a magnetic field, f being the intensity of the field at the point, and μ the permeability of the medium. This equation is derived from (18).

Proposition.—The rate, $\frac{dW}{dt}$, at which the energy of an electric tube increases is $\frac{dW}{dt} = E\frac{dQ}{dt}$, in which E is the electromotive force along the tube, and Q is the charge at one of its ends.

Proof.—Equations (17) and (19) express the total energy of such a tube. Imagine the electric field at every point in such a tube to be everywhere increased in intensity in a given ratio. Then N (and also Q), being the surface integral of ke (see equation (7)), and E, being the line integral of e (see equation (5)), will be both increased in that ratio. That is, E/Q remains constant, so that we may write

$$E = bQ \qquad (a)$$

in which b is a constant of which the value does not concern us. Substituting this value of E in (19), we have for the energy of the electric tube:

$$W = \tfrac{1}{2} bQ^2 \qquad (b)$$

whence

$$\frac{dW}{dt} = bQ\frac{dQ}{dt} \qquad (c)$$

Substituting E for bQ from (a), we have

$$\frac{dW}{dt} = E\frac{dQ}{dt}$$

Q. E. D.

Therefore we have the equations:

$$\frac{dW}{dt} = E\frac{dQ}{dt} \qquad (23)$$

in which $\frac{dW}{dt}$ is the rate of increase of the energy of an electric tube of strength Q, and E is the electromotive force along the tube. The tube may or may not be endless.

$$\frac{dW}{dt} = \Omega\frac{dm}{dt} \qquad (24)$$

in which $\frac{dW}{dt}$ is the rate of increase of the energy of a magnetic tube of strength m, and Ω is the magnetomotive force along the tube. The proof of this equation is identical to the proof of equation (23).

245. Isolated equations.—All the above equations occur in pairs. Every electric or magnetic equation which involves, directly or indirectly, the conception of *electric conduction*, stands by itself, for there is no such thing as *magnetic conduction*. Thus all equations which involve electric conduction-current, resistance, and electrostatic capacity, are isolated* equations.

Remark.—All equations which involve the rate of change, $\frac{dQ}{dt}$, of the strength of an electric tube, for example, equations (23), (27), (28) and (29), are identical to certain equations which involve *conduction-current* instead of $\frac{dQ}{dt}$. For example, equations (23) and (29) of this chapter are identical to equations (33), Chapter III., and (69), Chapter VIII., respectively. An electric tube, of which the strength is changing, is, therefore, equivalent, in its magnetic action, to a conduction current. This variation of an electric tube is therefore called a *displacement current* or a *dielectric current.*

246. Electromagnetic equations.—All phenomena which are associated in any way with the above equations are either wholly electric or wholly magnetic. The mutual dependence of electric and magnetic phenomena is a matter for distinct experimental discovery. There are a number of electromagnetic phenomena, and any one of these phenomena being established by experiment, all the others can be shown to be necessary, as will be

* The two equations, $\Phi = Li$ (57) and $Q = CE$ (89), are often given as a pair of electromagnetic equations, but they are analogous only in form. Physically, there is not the least similarity between them.

clearly seen in the following development. The phenomenon to be taken as fundamental is open to choice. We shall here choose the phenomenon of the production of electromotive force around a closed loop by a changing magnetic flux through the loop, from which we get the following:

247. Independent electromagnetic equation.

$$E = -\frac{dM}{dt} \tag{25}$$

That is to say, the electromotive force, E, around a closed loop, (in a homogeneous medium) is proportional * to the rate of change, $\frac{dM}{dt}$, of the magnetic flux through the loop. The minus sign is chosen for the reason that a positive value of $\frac{dM}{dt}$ gives a left-handed electromotive force around a loop.

248. Derived electromagnetic equations.—Consider an endless electric tube of strength Q linking with an endless magnetic tube of strength m (see Fig. 191). Let the positive directions around these tubes be chosen, as shown by the arrows. The electric flux through the tube Q is

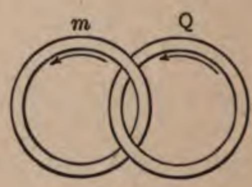


Fig. 191.

$$N = 4\pi Q \tag{a}$$

from equation (11); and the magnetic flux through the tube m is

$$M = 4\pi m \tag{b}$$

The electromotive force † around the electric tube is, by equation (25),

* The proportionality factor, which is found to be the same for all homogeneous media, may be considered to be unity, inasmuch as we already have more electrical quantities (by one) than independent electric and magnetic equations, and it would be meaningless to introduce another which always has the same value.

† That is, that *part* of the electromotive force which depends upon the changing strength of the magnetic tube. The electromotive force (total) around a closed curve is equal to the rate of change of flux (total) through the curve, so that each changing magnetic tube may be considered to produce a definite portion of the total electromo-

$$E = -\frac{dM}{dt} \tag{25 bis}$$

or, using equation (*b*),

$$E = -4\pi\frac{dm}{dt} \tag{26}$$

in which $\frac{dm}{dt}$ is the rate of change of the strength of the magnetic tube.

If the strength of the electric tube is changing, its energy will be changing at a rate $\frac{dW}{dt} = E\frac{dQ}{dt}$ from equation (23), or, substituting the value of E from (26), we have

$$\frac{dW}{dt} = -4\pi\frac{dm}{dt}\cdot\frac{dQ}{dt} \tag{27}$$

Therefore, the product $\frac{dm}{dt}\cdot\frac{dQ}{dt}$ being positive, *the electric tube is losing energy at the rate* $4\pi\frac{dm}{dt}\cdot\frac{dQ}{dt}$. This loss of energy by the electric tube is dependent upon the mutual action of the electric and magnetic tubes, so that the magnetic tube must be *gaining energy at the rate* $4\pi\frac{dm}{dt}\cdot\frac{dQ}{dt}$; that is,

$$\frac{dW}{dt} = +4\pi\frac{dm}{dt}\cdot\frac{dQ}{dt} \tag{28}$$

Comparing this with equation (24), we see that $+4\pi\frac{dQ}{dt}$ is a magnetomotive force around the magnetic tube; that is,

$$\Omega = +4\pi\frac{dQ}{dt} \tag{29}$$

or, using equation (*a*),

$$\Omega = +\frac{dN}{dt} \tag{30}$$

tive force. In the equations of this article, E is that *part* of the electromotive force around the tube Q due to the action of the given magnetic tube m; $\frac{dW}{dt}\left(=E\frac{dQ}{dt}\right)$ is that *part* of the rate of change of energy of the electrical tube which depends upon the action of the given magnetic tube, etc.

Remark.—Equations (26) to (30) are derived from equation (25); any one of these equations being considered as a result of experiment may be used as an *independent* equation in place of equation (25). Equation (27) is best suited to this purpose because of its symmetry.

249. Transformation of equation (27).—The differentials $\frac{dm}{dt}$ and $\frac{dQ}{dt}$ in equation (27) render its use as an independent equation, along with equations (1), (2), (3), (4), (5), (6), (7) and (8), somewhat obscure. For this reason it is convenient to write equation (27) in a form which does not involve differentials as follows: Imagine the rate of change $\frac{dm}{dt}$ of the magnetic tube to be uniform, m being the total change in time t, then $\frac{dm}{dt} = \frac{m}{t}$. Similarly imagine the rate of change $\frac{dQ}{dt}$ of the electric tube to be uniform, Q being the total change in time t, then $\frac{dQ}{dt} = \frac{Q}{t}$. Then $\frac{dW}{dt}$ will be uniform, and, W being the whole energy transformed in time t, we have $\frac{dW}{dt} = \frac{W}{t}$, so that equation (27) becomes $\frac{W}{t} = -4\pi\frac{mQ}{t^2}$, or,

$$W = -4\pi\frac{mQ}{t} \qquad (33)$$

This equation expresses the fact that if a magnetic tube of strength m be formed around a dielectric current of strength $\frac{Q}{t}$ in the positive direction, then an amount of electric energy $4\pi\frac{mQ}{t}$ will be transformed into magnetic energy.*

* Equation (33) is identical to the equation which expresses the work W done in carrying a magnetic pole of strength m around a conduction-current of strength $\frac{Q}{t}$. (See Article 124.)

250. Electromagnetic wave.—The equation (25) can be shown to be equivalent to

$$e = -\mu v f \tag{31}$$

in which f is the intensity of a magnetic field, which is moving through space at a velocity v at right angles to itself, and e is an electric field intensity, at right angles to both v and f, produced by the moving magnetic field. Compare Article 56, equation (39).

The equation (30) can be shown to be equivalent to

$$f = +kve \tag{32}$$

in which e is the intensity of an electric field, which is moving through space at a velocity v at right angles to itself, and f is a magnetic intensity at right angles to both e and v, produced by the moving electric field.

Consider the case of two moving fields, e and f, such that *each field is due to the motion of the other.* Then equations (31) and (32) are *simultaneous equations*, and give

$$v^2 = -\frac{1}{\mu k} \tag{33}$$

The velocity v is of course a vector, and its square is necessarily negative. Therefore, ignoring the negative sign, we have

$$v = \frac{1}{\sqrt{\mu k}} \tag{34}$$

Such mutually dependent moving electric and magnetic fields constitute an electromagnetic wave. The velocity of progression of such a wave is $\frac{1}{\sqrt{\mu k}}$ by equation (34). Various measurements by electrical methods of the quantity $\frac{1}{\sqrt{\mu k}}$ for air give for its value $298 \cdot 10^8 \frac{\text{cm}}{\text{sec}}$, which is *precisely the velocity of light in air.* Therefore, and for other reasons, it is thought that light consists of such electromagnetic waves. This is the so-called *electro-*

magnetic theory of light. The velocity of an electromagnetic wave, $v = \frac{1}{\sqrt{\mu k}}$, will be hereafter called simply the velocity of light.

251. Systems of electric and magnetic units.—We have the following * independent equations :

(1) $F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$ (3) $F = Qe$ (33) $W = 4\pi\frac{mQ}{t}$

(4) $F = mf$ and (2) $F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$

These five independent equations contain the six electric and magnetic quantities k, Q, e, f, m and μ. Therefore an arbitrary value must be assigned to some one of these quantities before the others are determined (or defined) by these equations.

252. "Electromagnetic" system of units.—The magnetic permeability μ of any given substance, say of air, may be taken as unity. Then,

$F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$ (2), defines strength of magnetic pole. (See Article 4.)

$F = mf$ (4), defines intensity of magnetic field. (See Article 6.)

$W = 4\pi\frac{mQ}{t}$ (33), defines electric current $\frac{Q}{t}$, or electric charge.†

$F = Qe$ (3), defines intensity of electric field. (See Article 158.)

$F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$ (1), defines specific inductive capacity.

* Aside from such equations as $E \equiv \Sigma e \cdot \cos \varepsilon \cdot \Delta s$, and aside from isolated equations. A given isolated equation is used to define the same electrical quantity in every case. For example, $Q = CE$ (89) defines electrostatic capacity C, when Q and E have been defined. The essential difference between the "electrostatic" and the "electromagnetic" systems of units is brought out most clearly by considering only those equations which are used to define different quantities in the respective systems.

† The equation which expresses Ampere's Law, viz., $F = Ilf$ (11) may be derived from the equation $W = 4\pi\frac{mQ}{t}$, so that the definition of electric current by $W = 4\pi\frac{mQ}{t}$ is essentially identical to the definition given in Article 30.

253. "Electrostatic" system of units.—The specific inductive capacity k of a given substance, say of air, may be taken as unity. Then,

$F = \frac{1}{k}\frac{Q_1 Q_{11}}{d^2}$ (1), defines electric charge (see Article 179), electric current being defined as unit charge per second.

$F = Qe$ (3), defines intensity of electric field. (See Article 158.)

$W = 4\pi\frac{mQ}{t}$ (33), defines strength of magnetic pole.

$F = mf$ (4), defines intensity of magnetic field. (See Article 6.)

$F = \frac{1}{\mu}\frac{m_1 m_{11}}{d^2}$ (2), defines magnetic permeability.

Remark 1.—Resistance, electromotive force, magnetomotive force, electrostatic capacity, self-induction, etc., are defined by the same equations in both systems, in terms, of course, of μ, m, f, e, Q, and k, as defined above for the respective systems.

Remark 2.—The words "*electromagnetic*" and "*electrostatic*," as used to designate the two systems of electric and magnetic units, do not signify anything. The systems ought perhaps to be called Weber's and Faraday's systems respectively.

254. Proposition.—*The number of "electrostatic" units of charge in one "electromagnetic" unit of charge is equal to the velocity of light in air.*

Proof.—The velocity of light in air is $v = \frac{1}{\sqrt{k\mu}}$, by Article 250. If an arbitrary value of unity be assigned to μ for air, then

$$v = \frac{1}{\sqrt{k}} \qquad (a)$$

Consider two equal charges Q. The force with which they repel at distance d is

$$F = \frac{1}{k}\frac{QQ}{d^2} \qquad (b)$$

or using $k = \frac{1}{v^2}$ from (a), we have

$$F = v^2 \frac{QQ}{d^2} \qquad (c)$$

Let Q' be the same charge expressed in "electrostatic" units ($k = 1$ for air), then the force of repulsion, which is of course the same as before, is

$$F = \frac{Q'Q'}{d^2} \qquad (d)$$

Comparing (c) and (d), we have

$$v = \frac{Q'}{Q} \qquad (e)$$

That is, the number Q', which expresses a given charge in "electrostatic" units, is v times as large as the number Q, which expresses the same charge in "electromagnetic" units. Therefore the "electromagnetic" unit of charge is v times as large as the "electrostatic" unit of charge. Q. E. D.

Similarly it may be shown that there are v "electromagnetic" units of magnetic pole in one "electrostatic" unit of magnetic pole.

Remark.—The equation $W = QE$ (15) expresses the work, W, required to carry a given charge, Q, along a given path in a given electric field, E being the electromotive force along the path. When the numerical value of Q (*given* charge) is large, the numerical value of E (given electromotive force) must be correspondingly small, since the product QE is the same no matter what units are used in expressing Q and E. Therefore there are $\frac{1}{v}$ "electrostatic" units of electromotive force in one "electromagnetic" unit of electromotive force.

The equation $Q = CE$ or $C = \frac{Q}{E}$ expresses the charge on a given condenser when charged with a given electromotive force. There are v "electrostatic" units of Q in one "electromagnetic" unit of Q, and $\frac{1}{v}$ "electrostatic" units of E in one "electromagnetic" unit of E, therefore there are v^2 "electrostatic" units of C in one "electromagnetic" unit of C.

The unit electric current is in every case a flow of one unit charge per second. Therefore there are v "electrostatic" units of current in one "electromagnetic" unit.

The equation $W = \frac{1}{2}Li^2$ expresses the kinetic energy, W, of a given current, i, in a given circuit of which the coefficient of self-induction is L. The product $\frac{1}{2}Li^2$ is therefore independent of the choice of electrical units, so that there must be $\frac{1}{v^2}$ "electrostatic" units of L in one "electromagnetic" unit.

In this way the following relations are easily verified:

Number of "electrostatic" units in one "electromagnetic" unit.

For Q, I, Ω, $f \cdots v$.

For m, E, e, $\cdots \frac{1}{v}$.

For C, k, and conductivity $\cdots v^2$.

For L, μ, and resistance $\cdots \frac{1}{v^2}$.

CHAPTER XVII.

DISTRIBUTED QUANTITY.*

255. A distributed scalar is a scalar quantity used to specify the condition or state of a medium. Such a quantity has, in general, a distinct value for each small part of the medium.

Homogeneous or *uniform* distribution occurs when the quantity has the same value throughout the medium; otherwise the distribution is said to be *non-homogeneous*.

Examples.—The temperature at each point of a body, the hydrostatic pressure at each point of a fluid, the density or mass per unit volume at each point of a substance, the electric charge per unit volume in a charged region, etc., are distributed scalars. The pressure in the atmosphere and the pressure in a liquid in a vessel are examples of non-homogeneously distributed pressure. The atmosphere furnishes an example of a non-homogeneously distributed density.

256. Volume integral of a distributed scalar.—Let V be the density of a body at a point, then $V \cdot \Delta\tau$ is the mass of material in the volume element $\Delta\tau$ at the point and the total mass M of the body is

$$M = \Sigma V \cdot \Delta\tau\dagger$$

* The object of this chapter is to explain in as simple a manner as possible the various geometrical notions which rise in the study of electricity and magnetism. The subject matter of the chapter constitutes a branch of pure geometry, a clear knowledge of which is absolutely essential to the study of Maxwell's Theory of Electricity and Magnetism. Such Cartesian expressions and developments as are given in this chapter are known to be true, but their use here is more of the nature of exposition than proof. The student who wishes to read extensively must be familiar with these Cartesian expressions.

† The student of physics must think in every case of an integration as a summation; the symbols Σ and $\int$ have to him the same meaning.

or

$$M = \int V \cdot d\tau \tag{124}$$

This summation, $\int V \cdot d\tau$, is called the *volume integral* of the distributed scalar V. The significance of volume integral is not in every case so simple as in case of density. If V is the volume density of electric charge then $\int V \cdot d\tau$ is the total electric charge in the region throughout which the summation is extended. If V is the energy per unit volume in an electric field, in a magnetic field, in a strained solid, or in a moving fluid, then $\int V \cdot d\tau$ is the total energy in the region of summation.

257. The gradient of a distributed scalar.—Let V be the value of a distributed scalar at a point p, and let $V + \Delta V$ be its value at an adjacent point distant Δx from p; then $\frac{\Delta V}{\Delta x} (= X)$ is called the *gradient* of V in the direction of Δx or in the direction of the x-axis of reference. We may therefore write

$$\begin{aligned} X &= \frac{dV}{dx} \\ Y &= \frac{dV}{dy} \\ Z &= \frac{dV}{dz} \end{aligned} \tag{125}$$

where X, Y, and Z are the components of a definite * vector called the *resultant gradient*, or simply *the gradient* of V at the point p. The gradient of a distributed scalar is therefore a distributed vector.

A scalar quantity distributed over a plane may be represented at each point of the plane by a height measured up from the plane. The complete distribution of the scalar quantity will then be represented by a raised surface, a hill, and the gradient of the scalar will be represented by the grade of the hill at each point.

Examples.—At a given point in a liquid the pressure is 15 units. At a point 20 cm. deeper the pressure is 75 units. The

* See "The Theory of Electricity and Magnetism," A. G. Webster, page 22.

difference in pressure, 60 units, divided by the distance, 20 cm., gives 3 units pressure per centimeter as the average pressure gradient between the two points. The temperature of one side of a wall is 2000° C. and the temperature of the other side is 200° C. The thickness of the wall is 30 cm. and the average temperature gradient through the wall is 60 degrees per centimeter.

258. A distributed vector is a vector quantity used to specify the condition or state of a medium. Such a quantity has, in general, a distinct value for each small part of the medium.

Homogeneous or *uniform* distribution occurs when the vector has the same magnitude and the same direction for all parts of the medium; otherwise the distribution is said to be *nonhomogeneous*.

Examples. (1) *Fluid motion.*—(*a*) The velocity at each point of a moving fluid is a distributed vector. (*b*) In certain cases of fluid motion each small portion of the fluid is rotating at a definite angular velocity about a definite axis. This rotary motion of the small parts of a moving fluid is a distributed vector. It is called *vortex motion*.

(2) *Gravitational field.*—A particle of matter, of mass m, at a given point in the neighborhood of the earth, is acted upon by the force $F = mg$. The quantity g has a definite magnitude and a definite direction (parallel to F) at each point in the neighborhood of the earth. It is called the intensity of the earth's gravitational field at the point.

(3) *Electric field. Magnetic field.*—The intensity at each point of an electric field is a distributed vector. The intensity at each point of a magnetic field is a distributed vector.

259. Stream lines of a distributed vector.—A line drawn through a region so as to be at each point in the direction of a distributed vector at that point, is called a *stream line* of the distributed vector. The manner of distribution of a vector is clearly represented by the use in imagination of such stream lines. In case of vortex motion these lines are called *vortex lines*. In case of electric field, of magnetic field, and of gravitational field these lines are called *lines of force*. The term *stream line* will be used in general statements.

A vortex line is, in accordance with the above definition, a line drawn through a moving fluid so as to be at each point parallel to the axis about which the fluid at that point is rotating. A familiar example of vortex motion is afforded by the ordinary smoke ring. The vortex lines in a smoke ring are a bundle of coaxial circles.

260. Permanent and varying states of vector distribution.—A distributed vector of which the value at each point is constant in magnitude and direction, is said to have a *permanent state* of distribution; otherwise the vector is said to be in a *varying state* of distribution.

Example.—If an orifice in a large tank of water is suddenly opened, a perceptible time elapses before the jet of water becomes established. During this interval the velocity of the water is changing rapidly at each point. After the jet becomes steady the velocity of the water at each point remains constant in magnitude and direction. The magnetic field in the neighborhood of a moving magnet, or in the neighborhood of a moving or changing electric current, is in a varying state. The electric field in the neighborhood of a moving charged body is in a varying state.

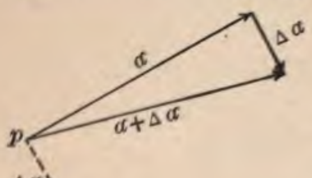


Fig. 192.

Rate of change of a distributed vector.—Let the line, α, Fig. 192, represent the value at a given instant of a distributed vector at the point p, and let the line $\alpha + \Delta\alpha$ represent the value of the vector after a time interval Δt has elapsed. Then $\Delta\alpha/\Delta t$ $(= \dot{\alpha})$, the rate of change of α, is a vector associated with the same point p, and its direction is, of course, parallel to $\Delta\alpha$. The vector $\dot{\alpha}$ is also a distributed vector having a definite value at every point.

The rate of change at each point of an electric field and the rate of change at each point of a magnetic field are very important considerations in Maxwell's theory of the electromagnetic field. The rate of change of electric field at a point is parallel to and proportional to what is called the *curl* of magnetic field at the same point; and the rate of change of magnetic field at a point is parallel to (but opposite in direction) and proportional to the *curl* of electric field at the same point. The concept *curl* is explained in a subsequent article.

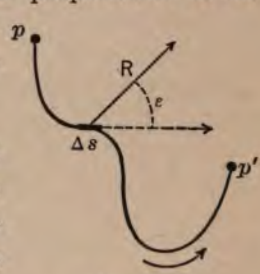


Fig. 193.

261. The line integral of a distributed vector.—Consider a line, pp', Fig. 193, in the region of a distributed vector. Let Δs be an element of this line, let R be the value of the distributed vector at the element, Δs, and let ϵ be the angle between R and Δs. Then $R \cos \epsilon \cdot$ is the resolved part of R parallel to Δs, $R \cos \epsilon \cdot \Delta s$ is the scalar part of the product of R and Δs,* and the summation

$$E = \Sigma R \cos \epsilon \cdot \Delta s$$

or

$$E = \int R \cos \epsilon \cdot \Delta s \tag{126}$$

is called the *line integral* of the distributed vector, R, along the line over which the summation is extended. The angle, ϵ, is reckoned between R and the positive direction of Δs, the positive direction of Δs being the direction in which Δs would be passed over in traveling along the line pp' in a chosen direction. If this chosen direction be changed, $\cos \epsilon$ will change sign at each element. Therefore, the line integral from p to p' is equal, but opposite in sign to the line integral from p' to p.

Examples.—The line integral of the velocity of a fluid is called the *circulation* of the fluid along the line over which the

* See Vol. I., Article 25. The product $R \cdot \Delta s$ is part vector and part scalar; so also is the sum $\Sigma R \cdot \Delta s$ or $\int R \cdot ds$; but the scalar part only is of great importance in the theory of electricity and magnetism.

integration is extended. The line integral of electric field along a line is called the *electromotive force* along the line. The line integral of magnetic field along a line is called the magnetomotive force along the line.

Cartesian expression for line integral.—Let X, Y and Z be the components of R, and let dx, dy and dz be the components of ds. Then the scalar part of the product $R \cdot ds$ is $X \cdot dx + Y \cdot dy + Z \cdot dz$, as is easily seen by multiplying the vector $X + Y + Z$ by the vector $dx + dy + dz$ and discarding such products as $X \cdot dy$ which, according to Article 25, Vol. I., are vectors. Therefore the line integral is

$$E = \int (X \cdot dx + Y \cdot dy + Z \cdot dz) \tag{127}$$

262. Proposition.—*The line integral of the gradient of a distributed scalar, V, along a line from p to p' is the difference in the values of V at p and at p' respectively.*

Proof.—Let R, Fig. 193, be the gradient of V at the element Δs. Then the resolved part, $R \cos \epsilon$, of R in the direction of Δs is, by Article 257, the gradient, $\frac{dV}{ds}$, in the direction of Δs. Therefore

$$R \cos \epsilon \cdot \Delta s = \frac{dV}{ds} \cdot \Delta s = \Delta V$$

so that

$$\Sigma R \cos \epsilon \cdot \Delta s = \Sigma \Delta V$$

which is evidently the total change in the value of V from p to p'.

Corollary.—(*a*) The line integral of the gradient of a distributed scalar is the same for all lines from p to p'; that is, the line integral of the gradient of a distributed scalar is independent of the path over which the integration is performed provided the ends of the path are fixed.

(*b*) The line integral of the gradient of a distributed scalar around a closed loop is zero.

Remark.—The line integral around a closed loop of the magnetic field in the neighborhood of an electric wire is zero provided the loop does not enclose the wire. When the path of integration circles round the wire Z times, the line integral of the magnetic field is $4\pi Zi$, where i is the strength of current in the wire. See Article 124.

Remark.—When X, Y and Z, equation (127), are the component gradients of a distributed scalar, V, then, as pointed out above,

$$X\cdot dx + Y\cdot dy + Z\cdot dz = dV$$

That is,

$$X\cdot dx + Y\cdot dy + Z\cdot dz$$

is the differential of a function V. Therefore,

$$\frac{dY}{dx} - \frac{dX}{dy} = 0 \quad \frac{dZ}{dy} - \frac{dY}{dz} = 0 \quad \text{and} \quad \frac{dX}{dz} - \frac{dZ}{dx} = 0$$

(See any good treatise on calculus.) These are the conditions which the components of a distributed vector must satisfy at each point of space, in order that the distributed vector may be looked upon as the gradient of a distributed scalar.

263. Irrotational vector distribution. Potential.—When a distributed vector satisfies the conditions above specified, it may be looked upon as the gradient of a distributed scalar and this distributed scalar is called its *potential.*

In case of fluid velocity the potential, if it exists, is called *velocity potential;* in case of gravitational field the potential is called *gravitational potential;* in case of electric field the potential, when it exists, is called *electric potential*, and in case of magnetic field the potential, when it exists, is called *magnetic potential.* When a distributed vector has a potential the distribution of the vector is said to be *irrotational*, for the reason that fluid velocity *has* a potential only in regions where there is no vortex motion.

Multivalued potential.—The magnetic potential in a region near an electric wire is multivalued. Imagine a plane drawn perpendicularly through the wire and consider the distribution of magnetic field in this plane. If this field is to be looked upon as a gradiant or, in concrete terms, as the grade of a raised surface, this surface will be similar to a winding stair surrounding the electric wire and having an infinite series of heights above a given point in the plane. These heights correspond to the possible values of the magnetic potential at the given point in the plane.

Remark.—The potential of a given distributed vector is a scalar whose *gradient* is everywhere fixed in value, and the value of the potential itself is indeterminate until an arbitrary value is assigned to it at some chosen place. This is always done by arbitrarily choosing the region of zero potential.

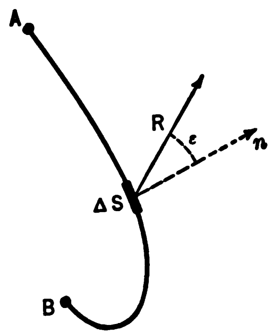


Fig. 194.

264. The surface integral of a distributed vector. Flux.—Let A and B, Fig. 194, be the points of intersection of a closed curve or loop, with the plane of the paper, and let the curved line, AB, represent a diaphragm to this loop. Let ΔS be the area of an element of the diaphragm, let R represent the value at ΔS of a distributed vector, and let ϵ be the angle between R and the normal to ΔS, this normal being always drawn out from the same side of the diaphragm. Then $R \cos \epsilon$ is the resolved part of R normal to ΔS, $R \cos \epsilon \cdot \Delta S$ is the scalar part of the product of R and ΔS,* and the summation

$$\Phi = \Sigma R \cos \epsilon \cdot \Delta S$$

or

$$\Phi = \int R \cos \epsilon \cdot dS \qquad (128)$$

is called the *surface integral* of the distributed vector, R, over that portion of the diaphragm over which the summation is extended. In case the normal be drawn to the other side of the diaphragm $\cos \epsilon$ will everywhere change in sign, and the surface integral, retaining its numerical value, will be changed in sign. In case

* See Vol. I., Article 25. The product $R \cdot \Delta S$ is part scalar and part vector; so also is the sum $\Sigma R \cdot \Delta S$ or $\int R \cdot dS$; but the scalar part, only, is of great importance in the theory of electricity and magnetism.

of integration over a closed surface the normal is understood to be drawn towards the interior always.

The surface integral of fluid velocity over a surface is the *flux* of fluid through the surface in cubic centimeters per second. The surface integral of any disturbed vector is called the *flux* of the vector. Thus we speak of the flux of magnetic field, the flux of electric field, etc.

In the case of a closed surface, we speak of the flux *into* or *out of* the region bounded by the surface.

265. Convergence of a distributed vector.—Consider a small region, of volume $\Delta\tau$, in the neigborhood of a point, p. Let $\Delta\Phi$ be the flux of a distributed vector, R, into this region. It can be shown when R is physically continuous, that the ratio $\frac{\Delta\Phi}{\Delta\tau}$ approaches a definite limiting value as the volume element, $\Delta\tau$, grows small. This limiting value of $\frac{\Delta\Phi}{\Delta\tau}$ is called the *convergence* of the vector, R, at the point p. From this definition we have

$$\Delta\Phi = \rho \cdot \Delta\tau \tag{129}$$

in which $\Delta\Phi$ is the flux of a distributed vector, R, into a small region, $\Delta\tau$, and ρ is the (mean) convergence of R in the region.

The convergence of a distributed vector is a distributed scalar. A negative convergence is sometimes called a *divergence*.

Cartesian expression for convergence.—Consider at a given point, p, a small cubical region of which the edges are Δx, Δy, and Δz. Let X, Y, and Z be the components of R at p. The flux of R across one of the $\Delta y \cdot \Delta z$ faces of the cube is $X \cdot \Delta y \cdot \Delta z$ *into* the region. Across the other $\Delta y \cdot \Delta z$ face the flux of R is

$$\left(X + \frac{dX}{dx} \cdot \Delta x\right) \Delta y \cdot \Delta z$$

out of the region. Therefore the total flux into the region across these two faces is

$$-\frac{dX}{dx} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

Similar expressions hold for the other two pairs of faces so that the total flux into the region is

$$-\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right)\Delta x\cdot\Delta y\cdot\Delta z$$

This quantity divided by the volume of the region $\Delta x\cdot\Delta y\cdot\Delta z$ gives the convergence ρ, so that

$$\rho=-\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right) \tag{130}$$

266. Breaking up of a surface integral over a closed surface into volume elements.—Consider the surface integral of a distributed vector, R, over a closed surface, normal directed inwards (compare Article 264). Imagine the enclosed region to be broken up into a large number of *cells*. *The surface integral of R over the closed surface is equal to the sum of the surface integrals of R over the enclosing surfaces of the various cells, normal directed inwards*, for any wall which separates two contiguous cells is integrated over twice with direction of normal reversed and the only surface integrals which are not cancelled in this way are the integrals over the various parts of the given closed surface.

Let $\Delta\Phi$ be the surface integral of R over one of the cells and let $\Sigma R\cos\epsilon\cdot\Delta S$ be the surface integral of R over the given closed surface. Then from the above statement we have

$$\Sigma R\cos\epsilon\cdot\Delta S=\Sigma\Delta\Phi$$

or, from equation (129),

$$\int R\cos\epsilon\cdot dS=\int\rho\cdot d\tau \tag{131}$$

That is, *the surface integral of a distributed vector, R, over a closed surface is equal to the volume integral of the convergence of R throughout the enclosed region.*

Examples.—The fact that the total electric flux *out of* a region is equal to 4π times the total amount of electric charge in the region shows that the convergence at a point of electric field is equal to *minus* 4π times the volume density of electric charge at the point.

267. Solenoidal vector distribution.—A distributed vector is said to have *solenoidal* distribution in a region throughout which its convergence is zero. The surface integral of such a distributed vector is zero over any closed surface.

Tube of flow.—Imagine stream lines to be drawn from each point in the periphery of a closed curve or loop. These stream lines form a tubular surface which is called a tube of flow of the given distributed vector.

Consider a number of diaphragms across a tube of flow; the flux is the same across them all, for any two diaphragms, together with the walls of the tube, constitute a closed surface into which the total flux is zero. The flux across the walls of the tube is zero, therefore, the flux into the enclosed space across one diaphragm is equal to flux out of the enclosed space across the other diaphragm.

Unit tube.—A tube of flow is called a unit tube when the flux through the tube is unity. For example, a unit tube has a sectional area of $\frac{1}{10}$ square centimeter in a magnetic field of 10 units intensity.

Imagine the entire solenoidal region of a distributed vector to be divided up into unit tubes. Then the flux across any surface anywhere in the region is equal to the number of these unit tubes which pass through the surface. Each unit tube may be conveniently represented in imagination by the single stream line along the axis of the tube. Then the flux across any surface in the region is equal to the number of these lines passing through the surface. In case of electric field and of magnetic field, the *lines of force* are always thought of as representing each a unit tube, and quantity of magnetic flux or of electric flux is expressed as so many lines of force.

Rotationally Distributed Vectors or Vectors which have no Scalar Potential.

268. Breaking up a line integral around a closed curve into surface elements.—Consider a distributed vector, of which the line integral around the closed (heavy) line, AB, Fig. 195, is not zero. Let the arrow represent the direction in which this line integral around AB is taken (compare Art. 261). *This line integral around AB is equal to the sum of the line integrals around the various meshes of any network, plane or otherwise, constructed in AB.* The line integrals around the various meshes are to be taken in the same direction as the line integral around AB, as shown by the two or three curled arrows.

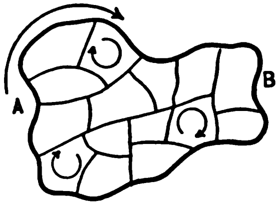


Fig. 195.

Proof.—Any line in the network not a portion of the heavy line is integrated over *once in each direction* in integrating around adjoining meshes; therefore (compare Art. 261), only the integrals along the portions of the heavy line are left outstanding.

Let ΔL be the line integral around one of the meshes in Fig. 195. Then, from the above, we have

$$\int R \cos \epsilon \cdot ds = \sum \Delta L \tag{132}$$

where $\int R \cos \epsilon \cdot ds$ is the line integral of R around AB.

This breaking up of the line integral around a closed curve into a number of elements, each of which refers to a small portion of a diaphragm (each mesh being a portion of a diaphragm to AB) to the closed curve, shows that the line integral around AB can be expressed as a surface integral over any diaphragm to AB.

269. Curl of a distributed vector.—Let ΔL be the line integral (a scalar) of the distributed vector, R, around one of the meshes, Fig. 195, of which the area is ΔS. The ratio $\frac{\Delta L}{\Delta S}$ approaches a definite limiting value as ΔS approaches zero. This limiting value of $\frac{\Delta L}{\Delta S}$ is the resolved part normal to ΔS of a definite* vector, C, which is called the *curl* of the distributed vector, R.

* See "The Theory of Electricity and Magnetism," A. G. Webster, pp. 53–57.

Let ϵ be the angle between C and the normal to ΔS. Then $\frac{\Delta L}{\Delta S} = C \cos \epsilon$ or $\Delta L = C \cos \epsilon \cdot \Delta S$, which, substituted in equation (132), gives

$$\int R \cos \epsilon \cdot \Delta s = \int C \cos \epsilon \cdot \Delta S$$

That is, *the line integral of a distributed vector R around a closed curve or a loop, is equal to the surface integral of its curl, C, over any diaphragm to the loop.*

Remark.—The surface integral of the curl C of a distributed vector is the same over any two diaphragms to a closed loop. Therefore the surface integral of C over any two diaphragms is zero if the direction of the normal is directed towards the interior of the region enclosed by the two diaphragms. Compare Article 264. That is, the surface integral of the curl of a distributed vector over *any closed surface is zero.* Therefore, according to Article 266 and equation (131), the *convergence of the curl* of any distributed vector is always and everywhere zero, that is, the vector C always has solenoidal distribution.

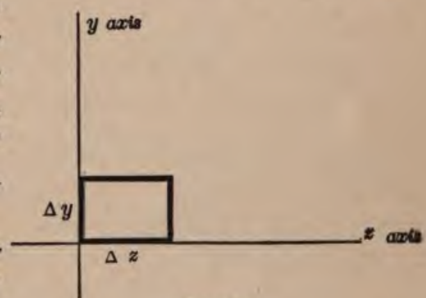


Fig. 196.

Cartesian expression for curl.—Consider the value of a distributed vector, R, in the neighborhood of a point. Let X, Y and Z be components of R. Consider the line integral of R around a small square whose sides are Δy and Δz, Fig. 196. The line integral of R along the side Δy is $Y \cdot \Delta y$; along the side parallel to this, the line integral is

$$-\left(Y + \frac{dY}{dz}\Delta z\right)\Delta y$$

The total line integral along these two sides is therefore

$$-\frac{dY}{dz} \cdot \Delta y \cdot \Delta z$$

Similarly the total line integral along the sides Δz is

$$+\frac{dZ}{dy}\cdot\Delta y\cdot\Delta z$$

So that the line integral around the square is

$$\left(\frac{dZ}{dy}-\frac{dY}{dz}\right)\Delta y\cdot\Delta z$$

which divided by the area $\Delta y\cdot\Delta z$ of the square gives

$$x\text{-component of curl of } R=\frac{dZ}{dy}-\frac{dY}{dz}$$

Similarly

$$y\text{-component of curl of } R=\frac{dX}{dz}-\frac{dZ}{dx}$$

and

$$z\text{-component of curl of } R=\frac{dY}{dx}-\frac{dX}{dy}$$

Examples of curl.—The angular velocity of a particle of a moving fluid is equal to one-half the curl of the velocity of the fluid. Electric current density, that is, the electric current per unit of sectional area of a wire is equal to $\frac{1}{4\pi}$ times the curl of the magnetic field. The factors *one-half* and $\frac{1}{4\pi}$ depend upon the choice of units.

The fact that the rate of change of magnetic flux through a loop is equal to the electromotive force around the loop shows that the rate of change of a magnetic field is equal to the curl of the electric field at the same point.

270. Vector Potential.—If a given distributed vector has no convergence it may be looked upon as the curl of another distributed vector. The new distributed vector, of which a given distributed vector is the curl, is called the *vector potential* of the given distributed vector.

PROBLEMS.

CHAPTER I.

1. Two round steel rods 1 cm. diameter and 25 cm. long are magnetized to an intensity of 600 units pole per square centimeter of sectional area. Calculate the strength of each pole. Calculate the force with which the north pole of one rod attracts the south pole of the other rod when the poles are at an approximate distance of 10 cm. from each other. Ans. 471.1 units pole, 2220 dynes.

2. Calculate the direction and intensity of the magnetic field, due to one of the magnets of problem 1, at a point distant 20 cm. from one pole and 15 cm. from the other pole. Ans. Intensity = 2.4 units.

3. Given three magnet poles A, B and C, for example the ends of three long, slim magnets. Poles A and B attract with a force of 200 dynes when they are 25 centimeters apart; B and C attract with a force of 150 dynes when they are 20 cm. apart; and C and A repel with a force of 120 dynes when they are 20 cm. apart. What is the strength of each pole? Ans. $A = \pm 316.2$ units pole, $B = \mp 395.3$ units pole, $C = \pm 151.8$ units pole.

4. Two magnet poles of equal strengths are at equal distances from a given point. Let f be the field intensity at the point due to one only of the poles. Give diagrams showing the relative positions of the two poles such that the resultant field at the point due to both poles may be equal numerically to f: (*a*) When the poles are alike, and (*b*) when the poles are unlike.

5. The intensity of the earth's magnetic field at Bethlehem is 0.57, and its dip is 63°. What are the vertical and horizontal components? Ans. $H = 0.259$, $V = 0.508$.

6. Find the direction and intensity of the resultant magnetic field at a point 22 cm. due magnetic north of an isolated pole of 572 units strength, at Bethlehem. Ans. 1.527 units intensity.

7. Find the distance and direction from the magnet pole of problem 6 to the point at which the resultant field is zero. Ans. 31.7 cm. south 63° up.

8. The moment of inertia of one of the magnets of problem 1 is 1,800 grams $\times$ cm^2. Calculate the periodic time of oscillation of this magnet when it is suspended horizontally at Bethlehem. Ans. 23.3 sec.

9. A magnet makes one complete oscillation per second in a magnetic field of 0.2 unit intensity. Another magnet twice as long, twice as wide, twice as thick, and magnetized to twice the intensity, is suspended in a field half as strong. What is its period of oscillation? Ans. $\sqrt{2}$ sec.

10. A suspended magnet makes 20 oscillations in 184.5 seconds at one place, and 20 oscillations in 215.8 seconds at another place. What is the ratio of the intensities of the horizontal component of the earth's magnetic field at the two places, and at which place is it the more intense? Ans. 1.367.

11. A horizontal suspended magnet is deflected by a bar magnet placed with its middle at a given distance due magnetic east of the suspended magnet as in Gauss's second arrangement. The deflection is 25° when the apparatus is set up in Bethlehem, and 32° when the apparatus is set up in New York. For the horizontal intensity of the earth's field at Bethlehem see problem 5. Required the horizontal intensity of the earth's field at New York. Ans. 0.193 unit.

12. One of the magnets of problem 1 is balanced horizontally on a knife edge at Bethlehem. The magnet weighs 125 grams. Find the horizontal distance from the knife edge to the center of the bar. Acceleration of gravity 980 cm. per sec^2. Ans. 0.049 cm.

13. A horizontal wire 200 cm. long, stretched due magnetic east and west, is moved vertically downwards over a distance of 150 cm. How many lines of force (that is, how much magnetic flux) does it cut at Bethlehem? How many lines of force pass through the floor of a room 10 meters wide and 30 meters long at Bethlehem? Ans. 7,770 lines, 1,524,000 lines.

14. The pole-face of a dynamo has an area 20 cm. × 30 cm. The magnetic field between the pole-face and the armature core is perpendicular to the pole-face at each point and its intensity is 6,000 units. Calculate the number of lines of force passing from the pole-face into the armature core. Ans. 3,600,000 lines.

15. How many lines of force pass out from a north pole of one of the magnets of problem 1? Ans. 5,915 lines.

CHAPTER II.

16. A horizontal wire 10 meters long, stretched due magnetic east and west, is pushed up by the horizontal component of the earth's field with a force of 2,500 dynes. What is the direction and strength of the current in the wire? The horizontal component of the earth's field being 0.2 unit. Ans. 125 amperes east.

17. The armature of a dynamo has a length, under the pole-face, of 30 cm. The magnetic field intensity between the pole-face and armature core is 6,000 units. The surface of the armature is covered with straight wires parallel to the axis of the armature. Each of these wires carries a current of 75 amperes. Calculate the force acting on each wire. Ans. 1,350,000 dynes.

18. A horizontal electric light wire stretched due magnetic north and south carries 1,000 amperes of current flowing towards the north. The length of the wire is 250 meters, the intensity of the earth's field is 0.57 unit and the magnetic dip is 63°. Find the value of the force pushing on the wire and specify its direction. Ans. 1,269,500 dynes west.

19. A circular coil of wire of 20 cm. radius has 15 turns of wire. How much current is required in the coil to produce at the center of the coil a field intensity of 0.57 unit? Ans. 0.121 c.g.s. units.

20. A circular coil of wire of 20 cm. radius has 15 turns of wire, and it carries a current of 20 amperes. Calculate the field intensity (*a*) at the center of the coil, (*b*) at a point in the axis of the coil distant 100 cm. from the center of the coil, and (*c*) at a point in the axis of the coil distant 200 cm. from the center of the coil. Ans. (*a*) 9.42, (*b*) 0.071, (*c*) 0.0093.

21. A tangent galvanometer gives a deflection of 10° for 1.2 amperes. Calculate the deflection which will be produced by 15 amperes. Ans. 65° 35′.

22. A rectangular frame 25 × 40 cm. has 10 turns of wire wound upon it. The frame is balanced horizontally upon an axis pointing due magnetic east and west. A current of 28 amperes is sent through the wire. Required the distance from the axis at which a 10-gram (9,800-dyne) weight must be hung to balance the torque action due to the earth's magnetic field at Bethlehem. (See problem 5.) Ans. 0.74 cm.

23. The spiral spring of a Siemens electrodynamometer is twisted through an angle of 225° to balance the force action on the movable coil when a current of 14 amperes flows through the instrument. A twist of 160° is required to balance the force action of a current which is being measured by the instrument Required the value of this current. Ans. 11.8 amperes.

24. A circular coil 10 cm. in diameter, having 50 turns of wire, is hung by a phosphor-bronze wire at the center of a large circular coil 120 cm. in diameter, having 500 turns of wire. The suspending wire is free from twist when the planes of the two coils are at right angles, and a torque of 250 dyne-centimeters twists the wire through one radian of angle. How much current must pass through the two coils in series to cause the suspended coil to turn 30° from its position of equilibrium? What happens if the current is reversed in one coil? What happens if the current is reversed in both coils? Ans. 0.27 ampere.

25. The horizontal component of the earth's magnetic field at the needle of a sensitive galvanometer (Kelvin type) is 0.18 unit, and its direction is due north. It is desired to produce at the needle a resultant magnetic field of 0.02 unit intensity in a due easterly direction. Find the distance and direction from the galvanometer needle at which an isolated north magnet pole of strength 600 units must be placed to produce the desired result. Ans. 57.6 cm., 6°20′ east of south.

CHAPTER III.

RESISTANCE.

26. A current of ½ ampere in a glow lamp generates 15 calories of heat in 10 seconds. Required the resistance of the lamp in ohms. What is the power expended in the lamp, in watts and in horsepower. Ans. 25.143 ohms, 6.3 watts, 0.0084 H. P.

27. A vessel contains 1,500 grams of water. When this vessel is heated and allowed to cool its temperature falls at the rate of 12° C. per minute as it passes the temperature of 90° C. A wire of 6 ohms resistance is submerged in the vessel. How much current must be sent through this wire to keep the temperature of the vessel at 90° C.? Ans. 14.5 amperes.

28. What is the resistance at 20° C. of one mile of a conductor consisting of seven pure copper wires, each 40 mils in diameter? Ans. 5.61 ohms.

Remark.—One *mil* is a thousandth of an inch. One *circular mil* is the area of a circle one mil in diameter. The area of a circle d mils in diameter is d^2 circular mils. When length is expressed in feet and sectional area in circular mils the numerical value of k equation (31) is 10.6 for pure commercial copper at 20° C.

29. Find the resistance at 20° C. of a copper conductor 100 feet long, having a rectangular section 0.5 × 0.25 inch. Ans. 0.00667 ohm.

30. A sample of commercial copper wire 3 feet long and 120 mils in diameter is found, by test, to have at the same temperature a resistance equal to that of 26.2 inches of pure copper wire 100 mils in diameter. Find the ratio of the specific resistance of the sample to the specific resistance of pure copper. Ans. 1.047.

31. What is the resistance at 20° C. of a steel rail 30 feet long weighing 900 lbs.? The specific gravity of the steel is 7.8, and its specific resistance at 20° is 6.8 times as great as the specific resistance of pure copper at 20° C. Ans. 0.0000753 ohm.

32. What is the resistance at 20° C. of an iron pipe 120 feet long having 1 inch inside diameter, and 1¼ inches outside diameter? The pipe has seven joints, and each joint is assumed to have the resistance of one foot of pipe. Specific resistance assumed to be the same as rail steel. Ans. 0.0154 ohm.

33. A pure copper wire 2,000 feet long weighs 125 lbs. What is its resistance at 20° C.? How will its resistance be changed by doubling its length without changing its weight? The specific gravity of copper is 8.9. Ans. 0.4043 ohm.

34. The specific resistance of carbon such as used for arc lamps is about 2,400 times as great as that of pure copper. Find the watts lost, that is, find Ri^2, in the two carbons of an arc lamp, 8 inches of each carbon being in circuit, the carbon being ½ inch in diameter, and the current passing through the lamp being 9.6 amperes. Ans. 12.5 watts.

35. A wire has a resistance of 164.8 ohms at 20° C. and a resistance of 186.2 ohms at 70° C. What is the temperature coefficient? Ans. 0.00274.

36. The field coil of a dynamo has a resistance of 42.6 ohms after the dynamo has stood for a long time in a room at 20° C. After several hours' run the resistance of the coil is 51.6 ohms. What is its temperature? Ans. 76.9°.

37. A platinum wire has 254 ohms resistance at 0° C. When placed in a furnace its resistance is 1,630 ohms. What is the temperature of the furnace? The temperature coefficient of pure platinum is 0.0037 per degree centigrade. Ans. 1463°.

38. A platinum wire which has 254 ohms resistance at 0° C. has a resistance of 81 ohms when placed in a bath of liquid air. What is the temperature of the liquid air. Ans. — 184.3°.

39. A glow lamp has a resistance of 220 ohms at a temperature of 1,000° C. (a bright red heat). At 20° C. its resistance is 277 ohms. What is the mean temperature coefficient of the carbon filament? Ans. — 0.000266.

ELECTROMOTIVE FORCE.

40. An electric generator produces 256 amperes of current in a circuit of which the resistance is 0.51 ohm. What is the electromotive force of the generator? How much current would this generator produce in a circuit of 2 ohms resistance if the electromotive force remains constant? How much power does the generator deliver to the 0.51-ohm circuit? to the 2-ohm circuit? Ans. 130 volts, 65 amperes, 34,380 watts, 8,450 watts.

41. Three coils having respectively 2 ohms, 3 ohms, and 5 ohms resistance, are connected in series between 110-volt mains. Calculate (*a*) the current flowing, (*b*) the electromotive force between the terminals of each coil, and (*c*) the power expended in each coil. Ans. (*a*) 11 amperes, (*b*) 22, 33, 55 volts, (*c*) 242, 363, 605 watts.

42. A coil of which the resistance is to be determined is connected between 110-volt mains, and the current is observed to be 26 amperes. What is the resistance? Ans. 4.23 ohms.

43. A battery of which the electromotive force is 1.07 volts and the resistance is 2.1 ohms is connected to a coil of 5 ohms resistance. What current is produced, and what is the electromotive force between the battery terminals? Ans. 0.15 ampere, 0.75 volt.

44. When electrical energy costs 15 cents per kilowatt-hour how much does it cost to operate, for ten hours, a glow lamp which takes ½ an ampere from 110-volt mains? Ans. 8¼ cents.

45. An electric motor which delivers 5 horsepower at its belt has an efficiency of 85 per cent. This motor is supplied with current from 110-volt mains. What current does it take? Ans. 43.88 amperes.

46. An electric generator having an electromotive force of 110 volts delivers 200 amperes current, which is transmitted to lamps 1,000 feet distant from the generator. Find the size of copper wire for mains in order that 95 per cent. of the power output of the generator may be delivered to the lamps, and find the electromotive force between the mains at the lamps. What size of copper wire would be required to deliver the same amount of power at the same distance with the same percentage loss, the electromotive force of the generator being 220 volts? What size to deliver the same amount of power with the same percentage loss to lamps 2,000 feet distant, the electromotive force of the generator being 110 volts? Ans. (*a*) 770,000 cir. m., 104.5 volts, (*b*) 192,000 cir. m., (*c*) 1,540,000 cir. m.

47. A motor using 100 kilowatts of power is 10 miles from the generator. Line wires 200 mils in diameter are to be used. What electromotive force is required at the generator in order that the line loss may be only 5 per cent. of the output of the generator? Ans. 7,675 volts.

48. What size of pure copper wire is required to deliver current at 110 volts to a 5-horsepower motor of 85 per cent. efficiency, the motor being 500 feet from the generator and the electromotive force of the generator being 115 volts? Ans. 84,800 cir. m.

SERIES AND PARALLEL CONNECTIONS.

49. The resistances of 2 and 5 ohms respectively are connected in parallel and included in a circuit in which a current of 12 amperes is flowing. Calculate the current in each coil, the electromotive force between the branch points, and the combined resistance of the two resistances. Ans. 8.57, 3.43 amperes, 17.14 volts, 1.43 ohms.

50. A telegraph line contains 200 miles of wire of 35 ohms per mile, 22 instruments each having 150 ohms resistance, and 200 cells of battery each having 2 ohms resistance—all in series. What is the total resistance of the line? Ans. 10,700 ohms.

51. A building contains 250 glow lamps each having 220 ohms resistance. The lamps are connected in parallel between mains of negligible resistance. What is the combined resistance of the circuit in the building? Ans. 0.88 ohm.

52. Six voltaic cells, each cell having 2 ohms resistance, are connected to a coil having 5 ohms resistance. What is the total resistance of the circuit: (*a*) when the six cells are connected in series? (*b*) when the cells are connected 2 in parallel and 3 in series? (*c*) when the cells are connected 3 in parallel and 2 in series? and (*d*) when the six cells are connected in parallel? Ans. (*a*) 17 ohms, (*b*) 8 ohms, (*c*) 6⅓ ohms, (*d*) 5⅓ ohms.

What current is produced in the coil in each case, the electromotive force of each cell of battery being 1.6 volts? Ans. (*a*) 0.56, (*b*) 0.6, (*c*) 0.5, (*d*) 0.3 ampere.

Remark.—When n voltaic cells are connected in series their combined electromotive force is nE where E is the electromotive force of one cell.

53. Three resistances A, B and C, of which the values are 500 ohms, 200 ohms, and 1.2 ohms, respectively, are connected to a battery of negligible resistance, the electromotive force of the battery being 2 volts. The connections are made so that the whole current produced by the battery flows through A, then divides and passes through B and C in parallel, and returns to the battery. Calculate the total resistance of the circuit, the total current, the current in B, the current in C, the electromotive force between the terminals of A, and the electromotive force between the terminals of B (or C). Ans. 501.19 ohms, 0.00399 ampere, 0.00025 ampere, 0.003965 ampere, 1.995 volts, 0.005 volt.

54. A direct-reading ammeter has a resistance of 0.03 ohm. The instrument is to be shunted so that the total current passing through the instrument and shunt is ten times the ammeter reading. What is the resistance of the shunt? Ans. 0.0033 ohm.

55. Three resistances of 4, 4, and 2 ohms respectively are connected in parallel; and two resistances of 6 and 3 ohms in parallel. The first combination is connected in series with the second, and with a battery of three volts electromotive force and negligible resistance. What is the current in the 2-ohm and 3-ohm resistances? Ans. 0.5 ampere, 0.66 ampere.

56. The scale of a direct-reading millivoltmeter has 150 divisions, each division corresponding to one millivolt. The instrument has a shunt connected to its terminals, and is used as an ammeter. What is the resistance of the shunt if each scale division corresponds to 2 amperes. Ans. 0.0005 ohm.

57. The millivoltmeter of problem 56 has a resistance of 16.8 ohms. What resistance must be connected in series with the instrument so that each division may correspond to 10 millivolts? What resistance must be connected in series with the instrument so that each division may correspond to one volt? Ans. 151.2 ohms, 16,783.2 ohms.

58. A voltmeter of which the resistance is 250 ohms is connected to a battery of which the resistance is 18 ohms, and the electromotive force is 1.9 volts. Find the reading of the voltmeter. Ans. 1.775 volts.

CHAPTER IV.

59. A vertical wire 3 meters long is moved sidewise, towards magnetic east or west, at a velocity of 25 meters per second. Find the electromotive force induced in the wire in volts, the horizontal component of the earth's field being 0.18 unit. Ans. 0.00135 volt.

60. The pole-face of a dynamo is 30 cm. long in the direction parallel to the axis of the armature, and the field intensity in the gap space between the pole-face and the armature core is 6,000 units. The wires on the armature are 12 centimeters from the axis of the armature, and the speed of the armature is 1,800 revolutions per minute. Find the electromotive force in volts induced in each armature wire (30 cm. in length) as it passes across the pole-face. Ans. 4.07 volts.

61. A coil of wire wound on one tooth of an alternator armature has 80 turns. When the tooth is squarely under a north pole of the field magnet the magnetic flux through the coil is + 1,800,000 lines. When the tooth is squarely under a south pole of the field magnet the magnetic flux through the coil is − 1,800,000 lines. The time required for the tooth to move from a north pole of the field magnet to an adjacent south pole is $\frac{1}{120}$ of a second. Calculate the average electromotive force induced in the coil during this time. Ans. 345 volts.

62. The ring armature of a direct-current bipolar dynamo has 260 turns of wire upon it, the armature is driven at a speed of 1,200 revolutions per minute, and the magnetic flux from a pole-face into the armature core is 2,200,000 lines. Calculate the electromotive force of the dynamo in volts. Ans. 114.4 volts.

63. The armature described in the above problem has upon it 500 feet of pure copper wire 325 mils in diameter. What is the resistance of the armature from brush to brush? Ans. 0.0125 ohm.

Remark.—In a bipolar dynamo the wire on the armature constitutes two paths between the brushes.

64. The core of an induction coil carries 100,000 lines of magnetic flux, when current is flowing through the primary coil. When the primary circuit is broken the flux in the core drops to 10,000 lines in 0.003 second. How many turns of wire are required in the secondary coil in order that an average electromotive force of 25,000 volts may be induced in this coil during the 0.003 second? Ans. 83,333 turns.

65. A transformer takes alternating current from supply mains at 1,100 volts and delivers current to service mains at 110 volts. The primary coil of the transformer has 560 turns of wire. How many turns of wire are there in the secondary coil? The transformer delivers 250 amperes to the service mains. How much current does it take from the supply mains? A usual allowance in transformer coils is 1,000 circular mils sectional area of wire for each ampere. Find size of wire used in primary coil and in secondary coil of the tranformer. Ans. 56 turns, 25 amperes, 25,000 cir. mils, 250,000 cir. mils.

CHAPTER V.

66. The atomic weights of three elements, (*L*), (*M*) and (*N*), are 107.7, 48.23 and 93.71, respectively, and their chlorids are (*L*)Cl, (*M*)Cl and (*N*)Cl. The electrochemical equivalent of (*L*) in grams per coulomb is 0.0001118. What are the electrochemical equivalents of (*M*) and (*N*)? Ans. 0.0000501, 0.0000972.

67. The atomic weights of three elements, (*L*), (*M*) and (*N*), are 54.97, 89.21 and 103.6, respectively, and their oxids are $(L)_2O_3$, $(M)_2O_3$ and $(N)_2O_3$. The electrochemical equivalent of (*L*) is 0.0001935. What are the electrochemical equivalents of (*M*) and (*N*)? Ans. 0.000314, 0.000365.

68. The atomic weights of three elements, (*L*), (*M*) and (*N*), are 63.18, 29.37 and 43.92, respectively, and their sulphates are $(L)_2SO_4$, $(M)SO_4$ and $(N)_23SO_4$. The electrochemical equivalent of (*L*) is 0.0003279. What are the electrochemical equivalents of (*M*) and (*N*)? Ans. 0.0000762, 0.000076.

69. An element has two oxids, $(L)_2O_3$ and $(L)O$. Its electrochemical equivalent in the first case is 0.000379. What is it in the second? Ans. 0.000568.

70. A column of a 15-per-cent. solution of $CuSO_4$, 1 meter long, having 1 square millimeter section, has a resistance of 260,000 ohms. An electrolytic cell of this solution has two flat electrodes, 30×30 cm., 2.5 cm. apart. Calculate the current due to 2 volts between electrodes, allowing 0.2 volt for polarization. Ans. 24.9 amperes.

71. The resistance of a $CuSO_4$ cell at 20° C. is 1.40 ohms; at 100° C. its resistance is 0.78 ohm. What is the temperature coefficient of the resistance? Ans. 0.005 negative.

72. Practically all the energy of reaction is represented in the electrical output of a Daniell cell. The electromotive force of a Daniell cell on open circuit is 1.07 volts. How many calories of heat are generated when one gram of powdered zinc is stirred into a solution of $CuSO_4$? Ans. 756 calories.

73. A battery cell which is free from local action gives a current of 1.5 amperes for 50 hours. Calculate number of grams of zinc consumed. Ans. 91 grams.

74. A single Grenet cell consumes 125 grams of zinc during the time that the current from the cell is depositing 80 grams of silver. What portion of the zinc is consumed by local action? Ans. 80 per cent.

75. The external electromotive force of a storage cell, while it is being charged, observed at equal intervals of time, is 2.05, 2.06, 2.08, 2.11, 2.15, 2.21, 2.29 volts, the current remaining constant. During discharge the external electromotive force is 2.07, 2.01, 1.97, 1.94, 1.92, 1.90 and 1.89, the current remaining the same as during the charging. What is the efficiency of the cell? Ans. 92 per cent.

Chapter VI.

76. A current which gives a deflection of 59.4° on a tangent galvanometer is observed to deposit 1.436 grams of copper in $20^m\ 45^s$. What is the reduction factor of the galvanometer? Ans. 2.082.

77. A current which gives a reading of 0.265 ampere on a milli-amperemeter is observed to deposit 0.2006 gram of silver in 10^m 30^s. What is the error of the ammeter reading? Ans. 0.02 ampere.

78. A standard resistance of $\frac{1}{1000}$ ohm is connected in a circuit. A millivoltmeter, connected to the terminals of the standard gives a reading 145.2. What is the value of the current? Ans. 145.2 amperes.

79. A tangent galvanometer having a resistance of 1.45 ohms gives a deflection of 54.8° when connected to a Daniell cell. The deflection is reduced to 41.6° when an additional resistance of 2 ohms is connected in circuit. What is the resistance of the Daniell cell? Ans. 1.95 ohms.

80. A current of 150 amperes, measured by an ammeter, is passed through a dynamo from brush to brush. The dynamo is not running. The electromotive force between the brushes is 4.2 volts and the electromotive force between each brush holder and the commutator bar upon which the brush rests is .96 volt. What is the resistance of the armature and what is the resistance of each brush? Ans. 0.0152 ohm, 0.0064 ohm.

81. All the ground connections on a 25-mile telegraph line are broken, one terminal of a voltmeter having a resistance of 16,250 ohms is connected to the line, the other terminal of the voltmeter is connected to one terminal of a 150-volt battery, and the other terminal of the battery is connected to ground. The voltmeter reads 6.2 volts. What is the insulation resistance of the entire line, and what is its insulation resistance per mile? Ans. 376,750 ohms, 9,418,750 ohms per mile.

82. (*a*) A very sensitive galvanometer having a resistance of 10,000 ohms is connected to two points 6 inches apart on a pure copper wire 40 mils in diameter. A current of 15 amperes sent through the wire gives a deflection of 16.8 scale divisions on the galvanometer. What amount of current is required to deflect the galvanometer one division? (The resistance of one mil-foot is 10.6 ohms.)

(*b*) One terminal of this galvanometer is connected to one terminal of a 200-volt battery, the other terminal of the battery connects to a metal vessel partly filled with water. Inside of the metal vessel stands a glass beaker partly filled with water, and the other terminal of the galvanometer dips into the water in the beaker. The lip of the beaker is kept carefully dry so that the only current that can flow is that which passes through the glass walls of the beaker from water to water. The galvanometer gives a steady deflection of 1.3 scale divisions. What is the electrical resistance of the glass walls of the beaker? Ans. (*a*) 0.000000295 ampere, (*b*) 520,990,000 ohms.

83. A high-resistance tangent galvanometer gives a deflection of 36.1° when connected to a Daniell cell of which the electromotive force is 1.07 volts, and a deflection of 56.3° when connected to a fresh Grenet cell. What is the electromotive force of the Grenet cell? Ans. 2.2 volts.

84. A dynamo delivers 100 amperes at 110 volts. What is its output in watts? Its efficiency is 90 per cent. What is its intake of power, in watts, from the engine which drives it? Ans. 11,000 watts, 12,222 watts.

85. A glow lamp has a resistance of 220 ohms. How much power is required to maintain a current of one-half ampere in the lamp? Ans. 55 watts.

Chapter VII.

86. The current in a circuit has a value of 26 amperes at a given instant. Three-hundredths of a second later the current is 10.3 amperes. What is the average rate of change of the current during the interval? Is this rate positive or negative? Ans. —523.3 amperes per second.

87. Calculate the kinetic energy in joules of a current of 160 amperes in a circuit having an inductance of 0.05 henry. Ans. 640 joules.

88. A battery having an electromotive force of 10 volts, and a resistance of 1 ohm, is connected to the primary of an induction coil. The primary has 1,000 turns, and a resistance of 4 ohms. What is the current in the primary when the magnetic flux in the core is increasing at the rate of 500,000 lines per second? Ans. 0.75 ampere.

89. An electromotive force of 25 volts is connected to a circuit of which the resistance is 0.6 ohm and the inductance is 0.05 henry. At what rate is the current increasing: (*a*) At the instant the electromotive force is connected to the circuit; (*b*) at the instant that the current reaches a value of 10 amperes; and (*c*) at the instant that the current reaches a value of 35 amperes? Ans. (*a*) 500 amperes per second, (*b*) 380 amperes per second, (*c*) 80 amperes per second.

90. A current has been left to die away in a circuit of 0.6 ohm resistance and 0.05 henry inductance. Find the rate of decrease of the current as it passes the values 100 amperes, 10 amperes, and one ampere. Ans. —1,200 amperes per second, —120 amperes per second, —12 amperes per second.

91. The choke coil of a lightning arrester has an inductance of 0.0002 henry, and its resistance is negligible. The sparking between the terminals of the coil at the time of a lightning stroke shows that the electromotive force between the terminals of the coil reaches a value of 60,000 volts. Calculate the rate of increase of current in the coil just before the formation of the spark. Ans. 300,000,000 amperes per second.

92. A coil of which the inductance is 0.035 henry has 1,500 turns of wire. Calculate the magnetic flux through the coil due to a current of 50 amperes in the coil, and calculate the flux through a mean turn. Ans. 175,000,000 lines, 116,666 lines.

93. A certain spool wound full of wire 0.1 cm. in diameter has an inductance of 0.08 henry. The same spool is wound full of wire 0.32 cm. in diameter. What is its inductance? Ans. 0.000762 henry.

94. Find the approximate inductance in henrys of a cylindrical coil 25 cm. long, 5 cm. mean diameter, wound with one layer of wire containing 150 turns. Ans. 0.00133 henry.

95. The coil specified in problem 94 has an auxiliary winding of 200 turns of wire at its middle. Find the mutual inductance of the two coils. Ans. 0.00029 henry.

96. The auxiliary winding, specified in problem 95, has an inductance of 0.002 henry. What is the inductance of the two coils connected in series? (There are two possible values according to the connections.) Ans. 0.00275 henry, 0.00391 henry.

97. A small circular coil of 10 turns, 2 cm. in diameter, is suspended at the center of a large circular coil, 30 cm. in diameter, having 20 turns. If the planes of the coils are perpendicular to each other and a current of 2 amperes flows in each, find the torque between them. Ans. 10.5 dyne centimeters.

98. A spool 5 times as large as the spool mentioned in problem 93, but similar in shape, is wound with wire 6 millimeters in diameter. What is its inductance? Ans. 0.197 henry.

99. The field winding of a dynamo has 50 ohms resistance and, approximately, 7.5 henrys of inductance. Assuming that the current grows in the coil in accordance with equation (54), calculate the time required for the current in the winding to reach 2 amperes when the winding is connected to a generator of which the electromotive force is 110 volts. Ans. 0.359 second.

Chapter VIII.

100. A tube 100 cm. long is wound with 5,000 turns of wire. A current of 2 amperes is sent through the winding. What is the intensity of the magnetic field inside of the coil? Ans. 125.664 c.g.s. units.

101. The field magnet of a dynamo is wound with 7,500 turns of wire. The winding is to produce a magnetomotive force of 12,000 c.g.s. units. How much current is required? Ans. 1.27 amperes.

102. A long iron rod, 10 square centimeters section, is magnetized to an intensity of 1,000 units pole per unit section when placed in a uniform magnetic field, of which the intensity is 25 units. How much magnetic flux passes out from the north pole of the rod because of the field due to the pole? How much flux passes out from this end of the rod because of the magnetizing field? What is the total flux through the rod? Ans. 125,664 lines, 250 lines, 125,914 lines.

103. Find the magnetomotive force across the air gap between pole face and armature core of a dynamo, the length of the air gap being 1.8 centimeters and the field intensity in the gap being 6,000 units. Express the result in c.g.s. units and in ampere turns. Ans. 10,800 c.g.s. units, 13,611 ampere turns.

104. Find the magnetomotive force between the ends of a straight path 200 cm. long and inclined at an angle of 60° with a uniform magnetic field of 3 units intensity. Ans. 300 c.g.s. units.

105. A magnetic flux of 1,500,000 lines is to be sent through a round wrought-iron rod. The flux density is to be 13,000 lines per square centimeter. What is the diameter of the rod? The same flux is to be sent through a round cast-iron rod. The flux density is to be 7,000 lines per square centimeter. What is the diameter of the rod? Ans. 6.06 cm., 8.26 cm.

106. A round iron rod 1 cm. diameter and 10 cm. long is magnetized to an intensity of 1,300 units pole per square centimeter section when it is placed in a region which but for the action of the poles of the rod would be a uniform magnetic field of 108.7 units intensity. Assuming the poles to be concentrated at the ends of the rod calculate the net magnetizing field at the middle of the rod. Ans. 27 units.

107. The wrought-iron core of a transformer has a peripheral length of 100 cm. and a sectional area of 25 square centimeters. A steady current of 4 amperes is sent through 500 turns of wire wound on the core. How much magnetic flux is produced? Ans. 368,125 lines.

108. How much current would be required in the winding specified in problem 107 to produce a magnetic flux of 400,000 lines through the core? Ans. 8.2 amperes.

109. The magnetic circuit of a dynamo consists of wrought iron, cast iron and air-gap. The wrought-iron portion of the circuit is 50 centimeters long and 120 square centimeters in section. The cast-iron portion is 40 centimeters long and 220 square centimeters in section, and the air portion is 2½ centimeters long and 300 square centimeters in section. How many ampere-turns are required to force 1,600,000 lines of force through this circuit? Ans. 12,100 ampere-turns.

110. How much flux would be forced through the magnetic circuit specified in problem 109 by 13,500 ampere-turns?

Remark.—This problem is to be solved by calculating a series of values of ampere-turns for various assigned values of flux. The desired result may then be interpolated from this table. Ans. 1,729,400 lines.

111. A long wrought-iron rod, 5cm. in diameter, projects part way into a long coil having 12 turns of wire per centimeter. Assuming that the poles of the rod are at its ends calculate the force in dynes with which the rod is pulled into the coil, when a current of two amperes flows through the coil.

Remark.—Find the strength of pole of the rod and calculate the force acting on this pole, using equation (2). Ans. 695,200 dynes.

112. How much work is done in magnetizing a round wrought-iron rod 2.5 cm. in diameter and 45 cm. long to a flux density of $B = 16{,}500$ lines per square centimeter? Ditto for cast-iron rod of same size to $B = 9{,}200$? Use table given in Article 131. Ans. 20,300,000 ergs, 11,320,000 ergs.

113. Using Steinmetz's formula [equation (81)], calculate the work done per cycle in magnetizing 1 cubic centimeter of annealed refined wrought iron between the limits $B = \pm 4{,}000$; and compare the result with value interpolated from Swinburne's values. See table given in Article 135. Ans. 1,160 ergs.

114. A test piece of wrought-iron wire is 400 cm. long and 1 cm. in diameter. It is placed in a vertical position with its upper end 300 cm. due magnetic east of a suspended magnet. The magnetizing coil on the specimen has 12 turns of wire per cm. length. A current of 3.33 amperes is sent through this coil. The deflection of the suspended magnet observed by a telescope and scale is 10.2 cm., the distance of the scale from the suspended magnet being 99.6 cm.; and the horizontal intensity of the earth's field at the suspended magnet is 0.20 c.g.s. units. Find intensity of magnetization, magnetic flux density and permeability of specimen. Ans. (*a*) 1,375 units pole per square centimeter, (*b*) 16,910 lines per square centimeter, (*c*) 344.

115. An iron rod, 1 square centimeter sectional area, has 50 turns of wire wound upon it and connected to a ballistic galvanometer. The total resistance of the B. G. circuit is 250 ohms. The B. G. gives a throw of 11 scale divisions when 0.000045 coulombs passes through it. What throw of B. G. is produced when flux density in the iron rod is changed suddenly from +8,000 to −5,000. Ans. 6.34 divisions.

Chapter IX.

116. A thermoelement of iron and copper, with both junctions at 10° C., has an electromotive force of 0.0000021 volt when one junction is heated $\frac{2}{10}$ of a degree. What is the thermoelectric power of iron-copper at 10° C.? What would the electromotive force of the element be when one junction is at 9.99° and the other junction at 10° C.? Ans. 0.0000105 volt per degree, 0.000000105 volt.

117. Find, from the diagram (Fig. 85), the electromotive force of an iron-copper thermoelement with one junction at −50° C. and the other junction at 100° C. Ans. 2,925 microvolts.

118. Find the electromotive force of an iron-cadmium thermoelement with one junction at 0° C. and the other junction at 250° C. Ans. 1,508 microvolts.

119. The thermoelectric power of two metals is zero at 280° C. and 20 microvolts per degree at 40° C. What is the electromotive force of a thermoelement of the two metals when one junction is at 280° C. and the other at 70° C.? Ans. 1,837 microvolts.

120. One junction of a thermoelement is kept at a temperature of 35° C. The temperature of the other junction is slowly raised, and the electromotive force of the element passes through zero when the temperature is 217° C. What is the (approximate) neutral temperature of the two metals of which the element is made? Ans. 126° C.

121. A thermoelement of German silver and copper is connected to a sensitive galvanometer. The resistance of the circuit is 124 ohms. One junction of the element is kept at 50° C. What is the temperature of the other junction when the galvanometer gives a deflection of one scale division. The galvanometer gives a deflection of 12.8 scale divisions for a current of 16.3×10^{-10} amperes. There are two temperatures, one above 50° and the other below 50°, which satisfy the conditions of the problem. Ans. 50.00045° C., 49.99955° C.

122. A thermoelement is to be used as a pyrometer. One junction, A, is kept at 20° C. When the other junction, B, is at the boiling-point of sulphur (448° C.) the element has an electromotive force of 2,200 microvolts. When the junction, B, is at the melting-point of silver (950° C.) the electromotive force is 3,800 microvolts. Calculate the values of a, b and c in equation (84). Ans. $(a) = 60.7557$, $(b) = 3.035$, $(c) = 0.000139$.

Chapter X.

123. During 0.03 second a charge of 15 coulombs passes through a circuit. What is the averge value of the current during this time? Ans. 500 amperes.

124. A ballistic galvanometer gives a throw of 26.2 scale divisions when 0.0023 coulomb of charge is passed through it.

What is the reduction factor of the instrument? Ans. 0.00008-778 coulomb per division.

125. A charge of 0.000000003 coulombs is drawn out of one tinfoil coating and forced into the other tinfoil coating of a Leyden jar by a battery of which the electromotive force is 1.07 volts. What is the capacity of the jar in microfarads? Ans. 0.0028 microfarad.

126. One mile of a submarine cable has an insulation resistance of 200,000 ohms. What is the insulation resistance of 100 miles of the cable? The capacity of one mile of the cable is 0.06 microfarad. What is the capacity of 100 miles of the cable? Ans. 2,000 ohms, 6 microfarads.

127. A condenser has a capacity of 2.3 microfarads when the dielectric is air. When the condenser is submerged in a certain kind of oil the capacity is 7.5 microfarads. What is the inductivity of the oil? Ans. 3.26.

128. A condenser is to be built up of sheets of tin foil 12 cm. × 15 cm. The overlapping portions of the sheets are 12 cm. × 12 cm. The sheets are separated by leaves of mica 0.05 cm. thick. How many mica leaves and how many tinfoil sheets are required for a one-microfarad condenser? Assume the inductivity of the mica to be equal to 6. Ans. mica, 654, tin foil, 655.

129. A standard one-microfarad condenser is charged with a standard Clark cell (electromotive force equal to 1.434 volts). The condenser is then discharged through a ballistic galvanometer producing a throw of 23.6 scale divisions. What is the reduction factor of the ballistic galvanometer? Ans. 0.0000000-6076 coulomb per division.

130. A condenser of unknown capacity is charged with an electromotive force of 14.34 volts and discharged through the ballistic galvanometer specified in problem 129. The throw of the ballistic galvanometer is observed to be 5.3 scale divisions. What is the capacity of the condenser? Ans. 0.02246 microfarad.

131. A condenser is made of two flat metal plates separated by air. Its capacity is 0.003 microfarad. Another condenser has plates twice as wide and twice as long. These plates are separated by a plate of glass (inductivity 5) which is four times as thick as the air space in the first condenser. What is the capacity of the second condenser? Ans. 0.015 microfarad.

132. Two metal plates, 100 cm. × 100 cm., are separated by 2 cm. of air. This condenser is charged by a battery having an electromotive force of 2,000 volts. What is its energy in joules? Ans. 0.000885 joule.

133. A flat glass plate, inductivity 5, size 100 cm. × 100 cm. × 2 cm., is slid between the metal plates specified in problem 132, the battery being left connected to the metal plates. What is the energy of the condenser after the glass is in place? Ans. 0.004425 joule.

134. The 2,000-volt battery is disconnected from the metal plates specified in problem 133 after the glass is in place and the metal plates are thoroughly insulated. The glass plate is then withdrawn, the whole charge being left on the metal plates. What is the electromotive force between the metal plates after the glass plate is withdrawn? What is the energy of the condenser after the glass plate is withdrawn? How much has the energy been increased by withdrawing the glass? How much force was necessary to withdraw the glass, ignoring friction, weight, etc.? (Assume that the glass is withdrawn sidewise, not cornerwise.) Ans. 10,000 volts, 0.0221 joule, 0.0177 joule, 1770 dynes.

135. The air condenser specified in problem 132 is charged with 2,000 volts, the battery is then disconnected and the plates are then moved to a distance 3 cm. apart, charge on the plates remaining unchanged. What is the electromotive force between the plates after the movement? What is the increase of energy due to the movement? How much force was necessary to produce the movement, ignoring friction, weight, etc.? Ans. 3,000 volts, 0.0004425 joule, 4,425 dynes.

CHAPTER XI.

136. What is the intensity of the electric field between two parallel metal plates 15 cm. apart, the electromotive force between the plates being 25,000 volts? Ans. 1,666 volts per centimeter.

137. A charged metal ball is placed in a uniform electric field of intensity 1,500 volts per centimeter. The charged ball is acted upon by a force of 25 dynes. What is the charge on the ball in coulombs? Reduce all data to c.g.s. units to avoid confusion. Ans. 0.0000000000166 coulomb.

138. A straight line 100 cm. long makes an angle of 60° with a uniform electric field, of which the intensity is 1,000 volts per centimeter. What is the electromotive force between the ends of the line? Ans. 50,000 volts.

139. The contact electromotive force between two given metals is 1.3 volts. Flat plates of these metals 0.01 cm. apart are connected by a wire, then disconnected, insulated, and separated to a distance of 10 centimeters. What is the electromotive force between them? Ans. 1,300 volts.

140. The positive terminal of a 110-volt dynamo is chosen as the region of zero potential. What is then the potential of the other terminal? A 20-ohm coil and a 30-ohm coil are connected in series between the dynamo terminals, the 20-ohm coil being adjacent to the positive terminal. What is the potential of the junction of the two coils? Ans. −110 volts, −44 volts.

141. A twenty-volt battery sends current through a long uniform wire. The middle point of the wire is chosen as the region of zero potential. What is the potential of the zinc electrode of the battery; of the carbon electrode of the battery? Ans −10 volts, +10 volts.

CHAPTER XII.

142. Using the coulomb as the unit charge and one volt per centimeter as the unit of electric field intensity find the number of lines of electric flux emanating from unit electric charge. Ans. 11,230,000,000,000 lines.

Suggestion.—Assume parallel-plate air condenser of given dimensions. Calculate its capacity, using equation (92). Calculate charge q upon one of its plates with any assumed electromotive force, E. Then $E \div$ distance between plates is the electric field intensity between the plates. This multiplied by the area of one of the plates gives the elective flux Φ from plate to plate. This divided by q gives the flux per unit charge.

143. Calculate the capacity in centimeters (Faraday units) of a condenser consisting of a metal sphere 20 cm. diameter inside of and concentric with a hollow metal sphere 25 cm. inside diameter, and reduce the result to microfarads. Ans. 100 centimeters, 0.00111 microfarad.

144. An electromotive force to be measured is connected between the attracting plates of an absolute electrometer. The distance between the plates is 2 cm., the area of the movable plate is 60 square centimeters, and the attraction is counterpoised by 2 grams weight (1 gram = 980 dynes). What is the value of the electromotive force? Reduce to volts. Ans. 58.4 faraday units, 17,520 volts.

145. A quadrant electrometer connected as shown in Fig. 120, is observed to give a deflection of 10 divisions for $e = 20$ volts. The electrometer is then connected as shown in Fig. 121. The electromotive force E, Fig. 121, is 1,000 volts. What value of e will be required to produce a deflection of ten divisions? Ans. 0.6 volt.

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